# SOME EXPERIMENTAL RESULTS OF CORONA SHAPES OF PERIODIC AND APERIODIC TILINGS

Katsunobu Imai (Hiroshima University)

Oct. 5, 2018, Hiroshima

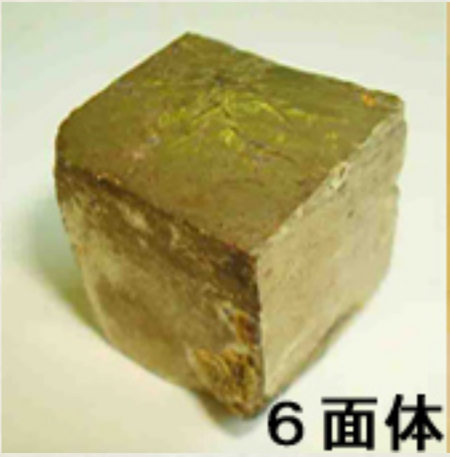
# CRYSTAL GROWTH

e.g. Pyrite

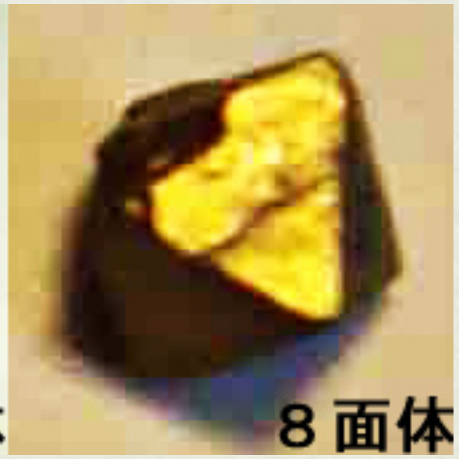
# DIFFERENT GROWTH FORMS

A final growth form depends on its environment.

## Pyrite




Hexahedron




Octahedron

## Garnet


Dodecahedron


Trisoctahedron

http://www2.city.kurashiki.okayama.jp/musnat/geology/mineral-rock-sirabekata/mineral44/mineral-jikeikesshou/mineral-jikeikesshou.html

# Isometric Array Grammar

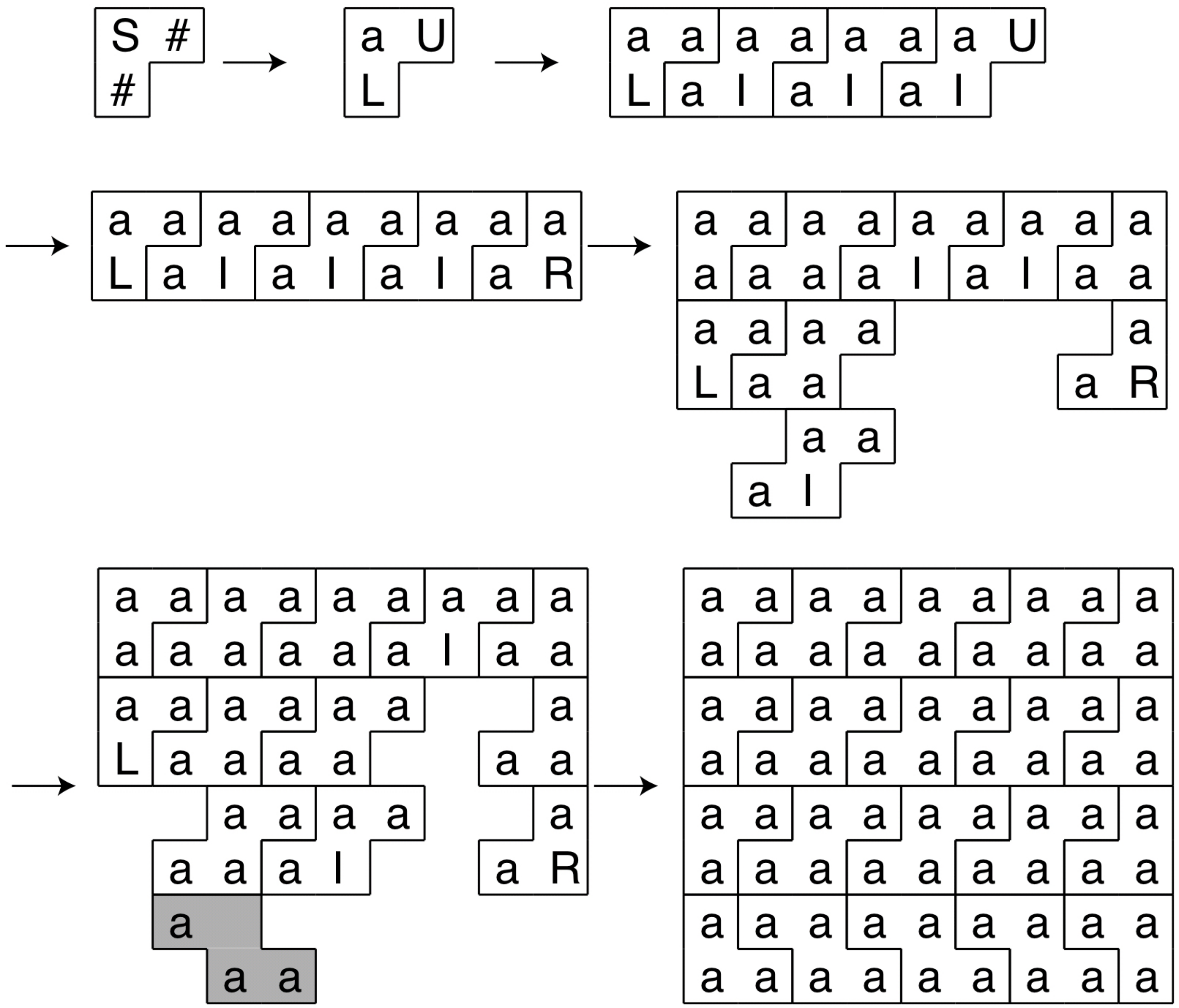


Fig. 4. A derivation example of $G_R$

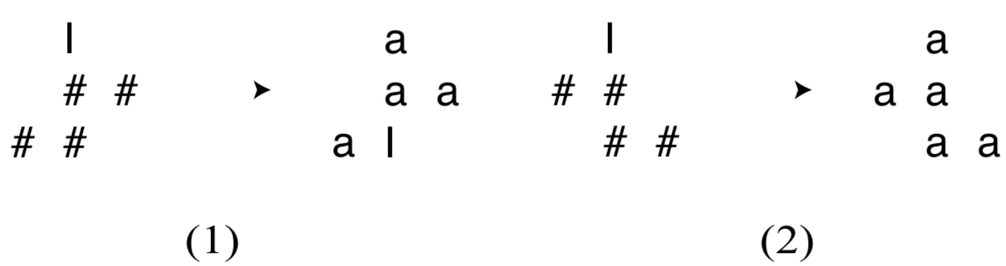
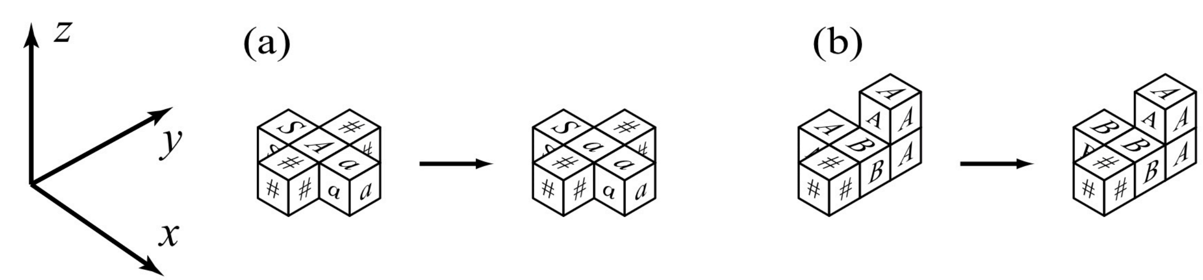


Fig. 5. The 'advancing' rule and the 'terminating' rule in $G_R$.

## Three-dimensional case:

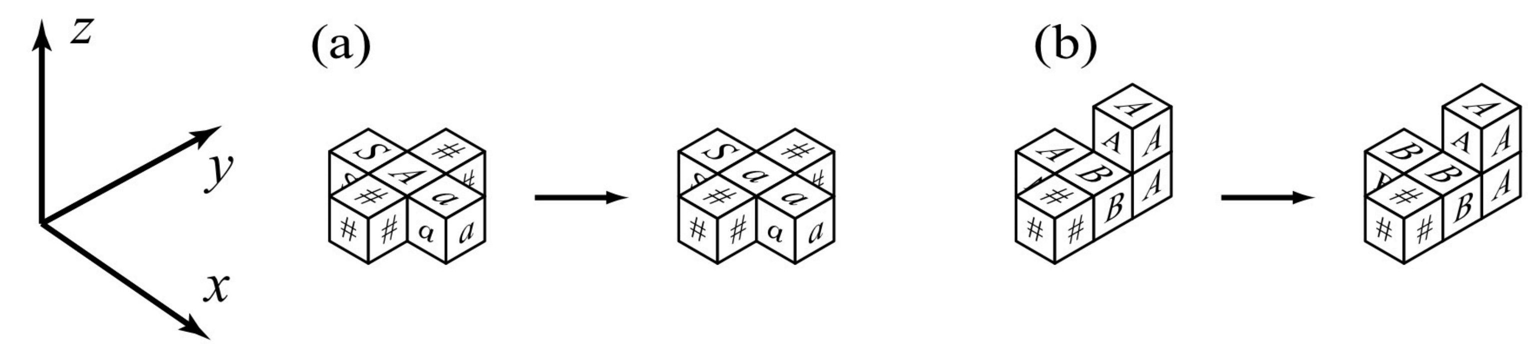


A context-free array grammar for cuboids (may be fail)

# An isometric context-sensitive array grammar

$G_{cube}$ that generates any cubes of symbol '$a$'s.

$$G_{cube} = (N_{cube},\ T_{cube},\ P_{cube},\ S,\ \#),$$
$$N_{cube} = \{S, X, Y, Z, F, A, D, G, E\},$$
$$T_{cube} = \{a\},$$



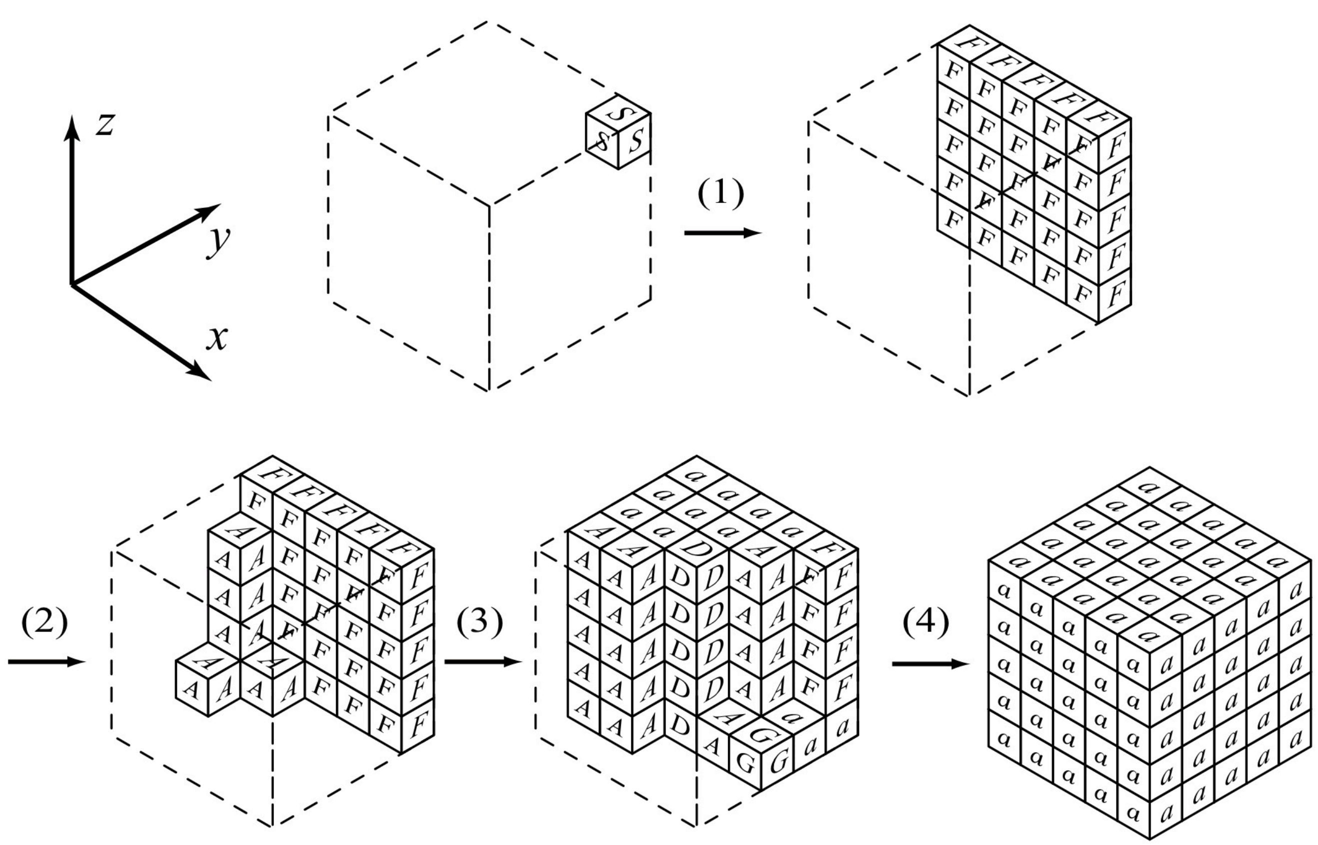
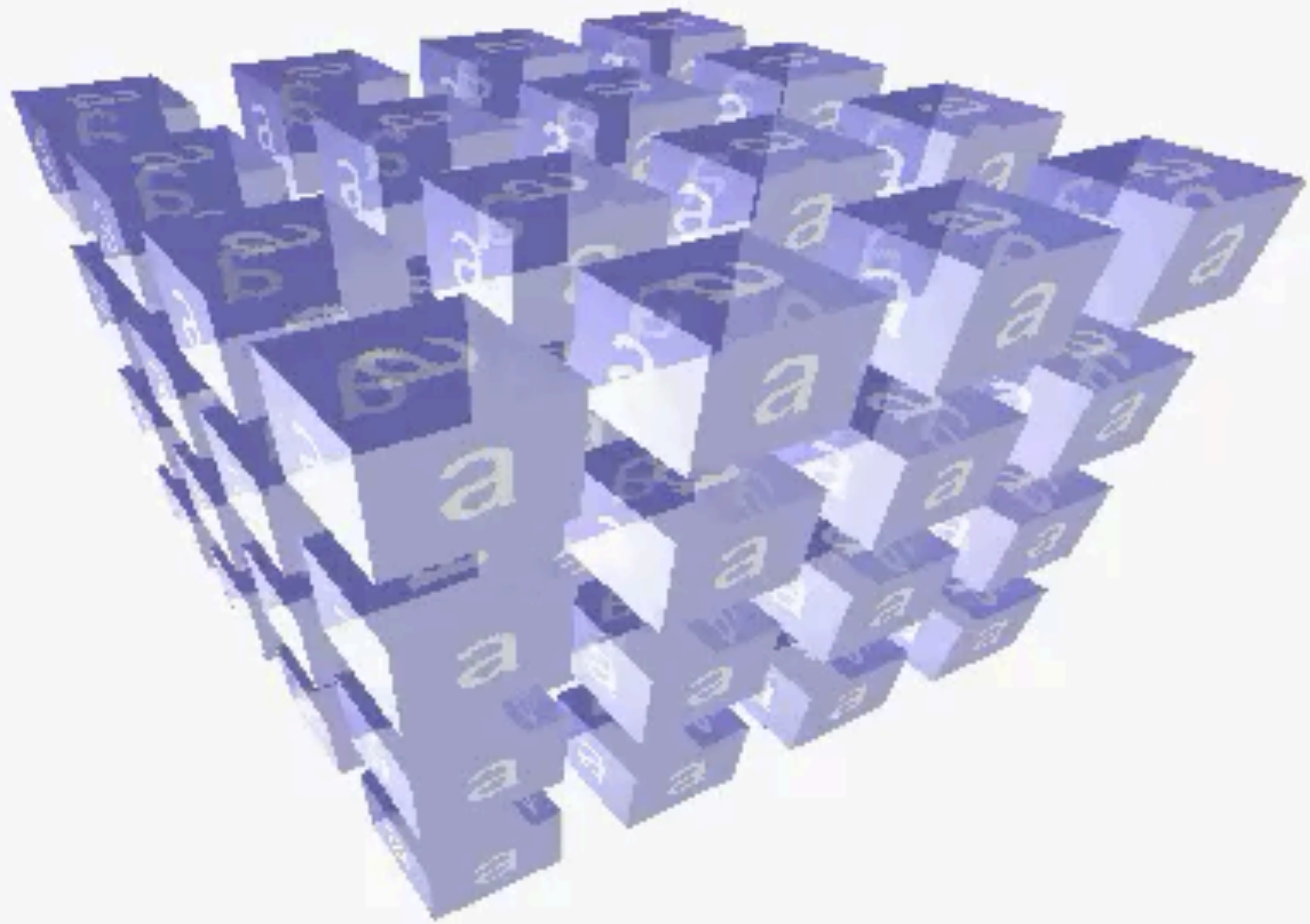




Fig. 8. The outline of derivation of a cube by $G_{cube}$

Imai, Matsuda, Iwamoto, Morita 2001

# A context-sensitive array grammar for cubes (no failure)



# Our nature behaves quite synchronizing, when an initial condition is homogenous.

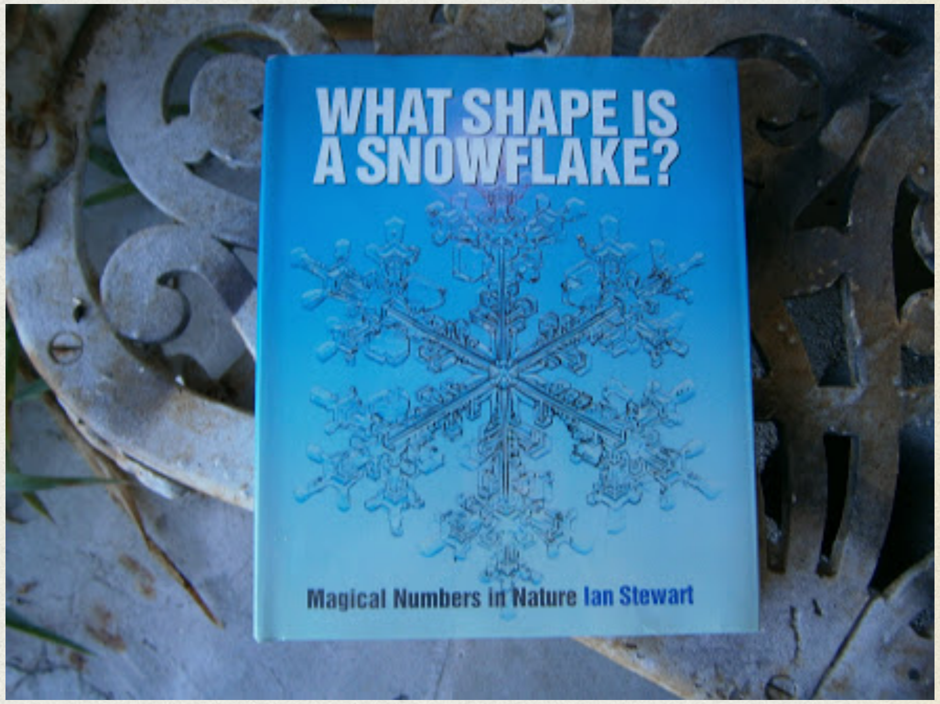
http://www.treasuremountainmining.com/index.php?route=pavblog/blog&id=73

# CRYSTAL GROWTH IS VERY DIFFICULT!

What is the growth of a snow crystal?

Ian Stewart (2001)

- Is it a kind of phase transition process? Yes.
- Is it a bifurcation? Yes.
- Is it a symmetry breaking? Yes.
- Is it a chaos? Yes.
- Is it a fractal? Yes.
- Is it a complex system? Yes.




- Can we completely understand it in future? **No**.

# CRYSTAL GROWTH

- It can be regarded as the movement of a solid-liquid (gas) interface.

- Its driving force is defined by the difference between liquid and solid (gas) chemical potentials at the interface per a small distance.

cf. phase field model

# LIMIT SHAPES OF CRYSTAL GROWTH

There are two stages of studies according to the macro scale shapes of crystals:

- <u>Equilibrium form</u> (at a closed env. with finite atoms)
  - defined by the final form as the result that a crystal growing system achieves its smallest free energy.
- <u>Growth form</u> (at an open env. with infinite atoms)

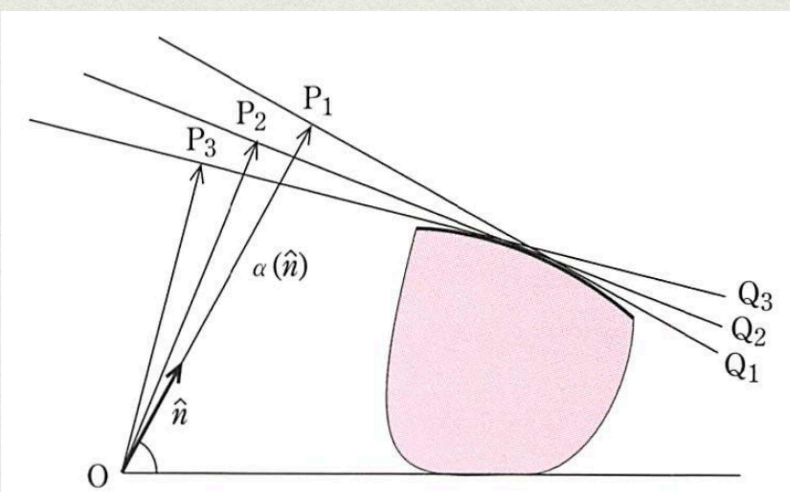
# WULFF PLOT OF EQUILIBRIUM FORM

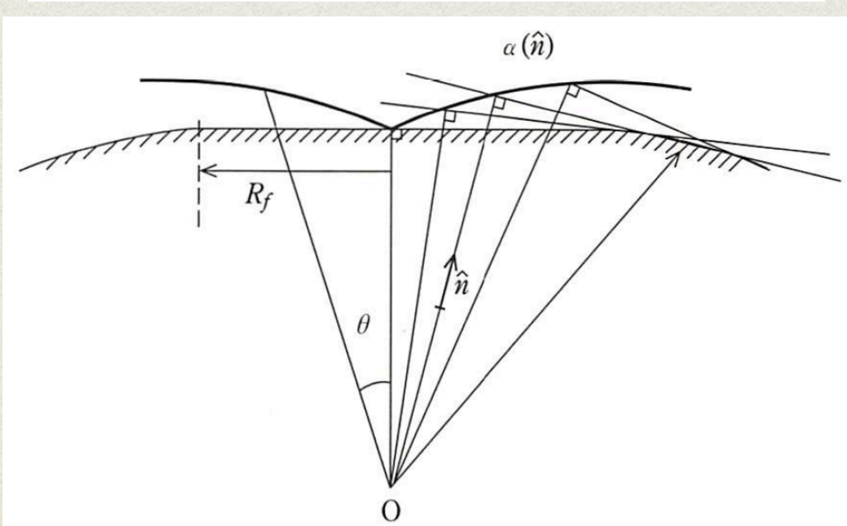
(Wulff 1901)

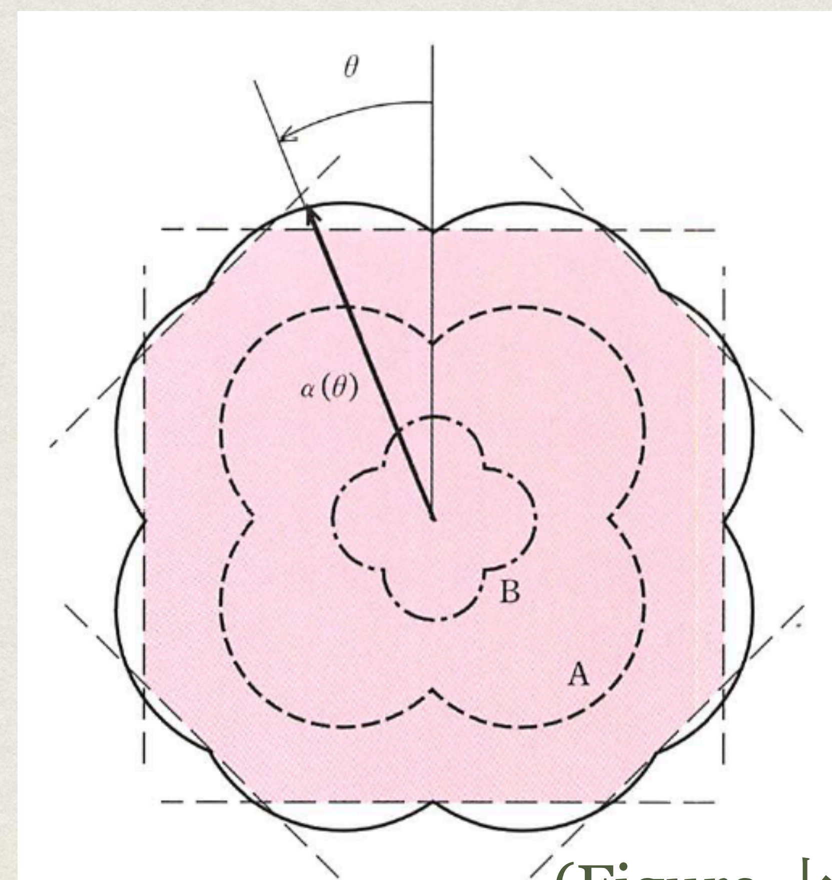
- If its **surface free energy density** α(θ) for each direction θ is known, the equilibrium form can be computed.

Ex. Primitive cubic lattice, low temperature

(i.e., ignoring the effect of entropy).



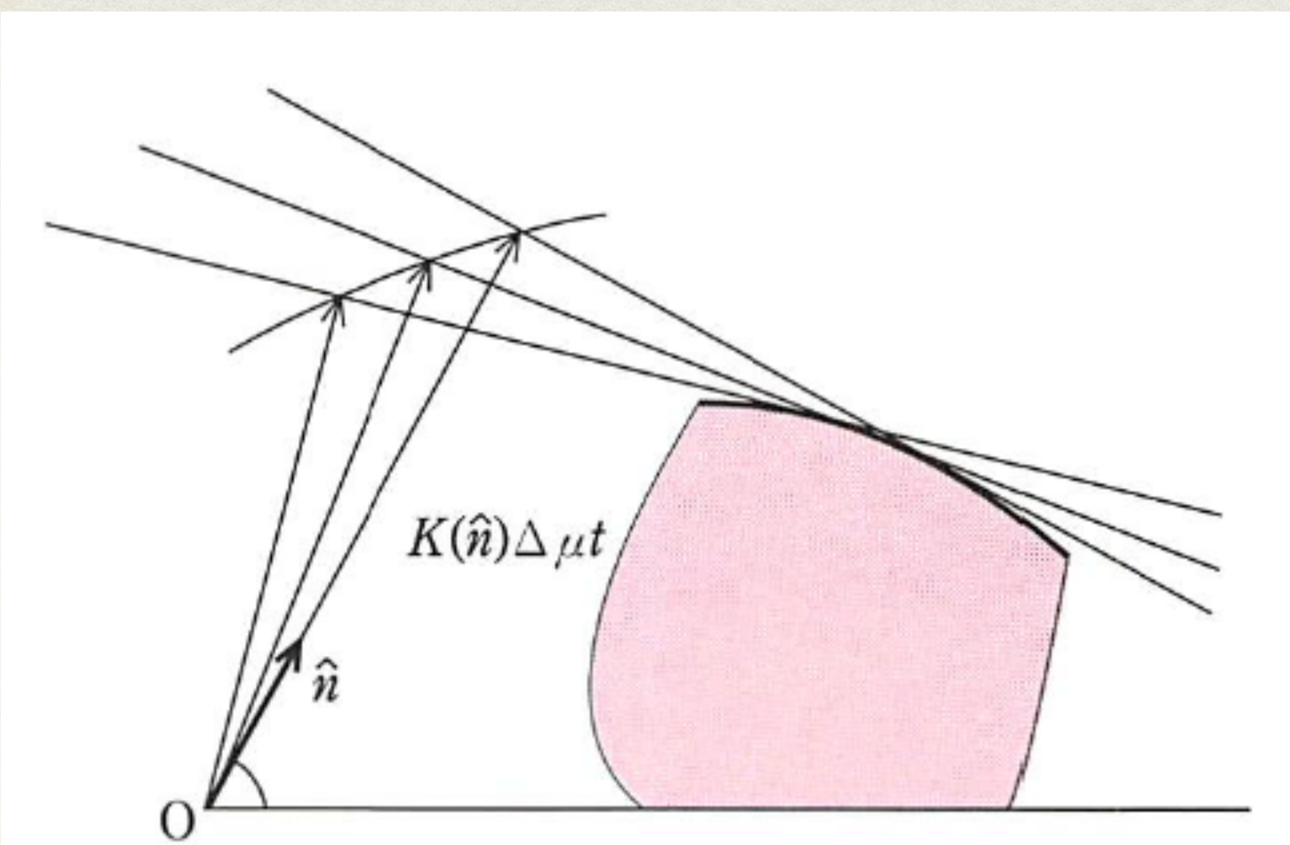

(Figure 上羽 2002)

# GROWTH FORM

(Chernov 1962)

- The asymptotical form of a crystal is similarly enlarged and the form is independent from the shape of initial nuclei. If the **velocity coefficient** $K(\theta)$ is known, the asymptotical growth form can be computed with the same way of Wulff plot.

$$v(\hat{n}) = K(\hat{n})\Delta\mu$$



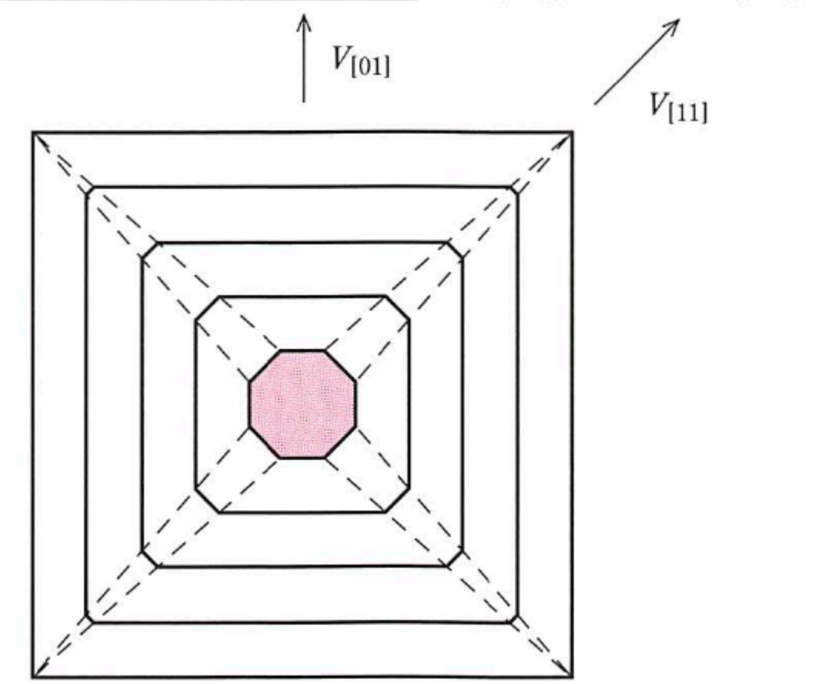




図 5.9　8 角形から出発した結晶の形の変化

$v_{[11]} = 1.5v_{[01]}$ とした。$v_{[11]} > \sqrt{2}v_{[01]}$ なので [11] 面は，いつかは消失する

(図 上羽 2002)

# LATTICE BASED PREDICTION OF GROWTH FORM

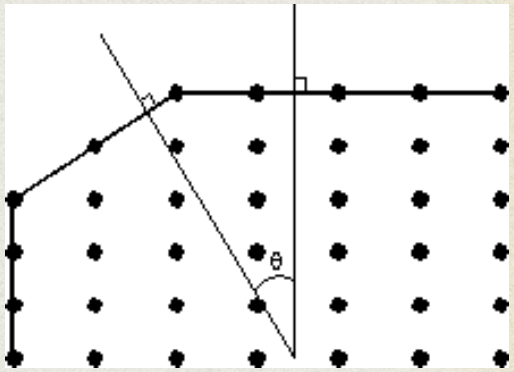
**Bravais' law**

"the faces most likely to be found on a crystal are those parallel to lattice planes of highest reticular density"

It relies on the 32 point groups.

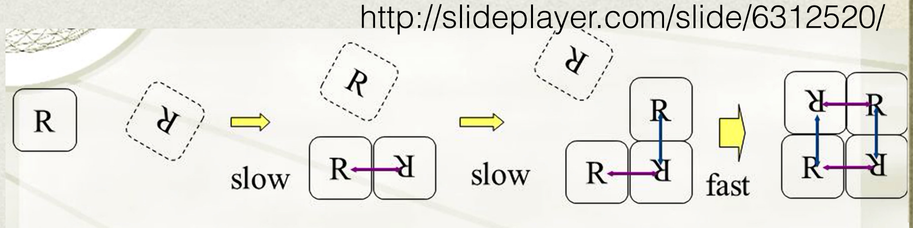
It is extended to the 230 space groups.

Donnay, Harker 1937



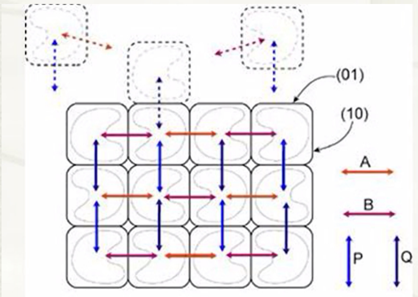
https://www.tulane.edu/~sanelson/eens211/crystalmorphology&symmetry.htm

http://slideplayer.com/slide/6312520/




To introduce the effect of energy,

**Periodic bond chains (PBCs)** theory focused on crystal zones and classified crystal faces according to the number of periodic bond chains.




Hartman and Perdok 1956

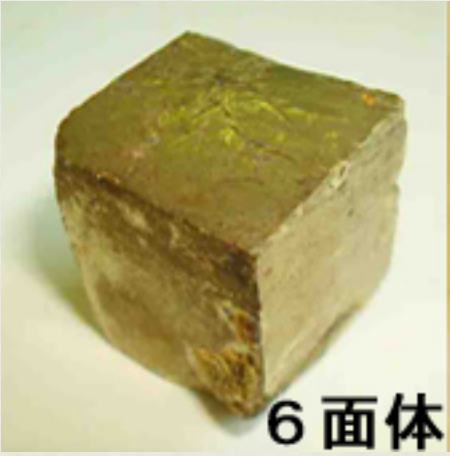
We would like to consider about the ideal case of growth forms of a single crystal from a single nucleus in a homogeneous environment.

Possible growth from:

should be governed by each tiling

should have some variants for each tiling

Pyrite




Hexahedron


Octahedron

Garnet


Dodecahedron


Trisoctahedron

# CORONA

## as a very simple model of crystal growth.

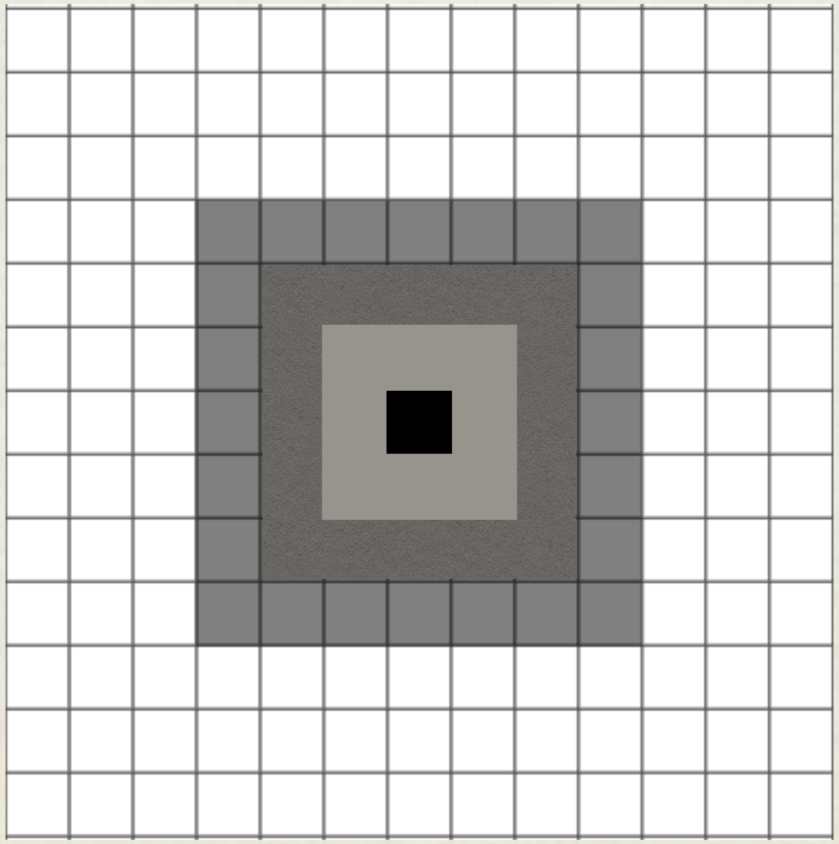
**Just kidding! It must be extraordinarily complex!**

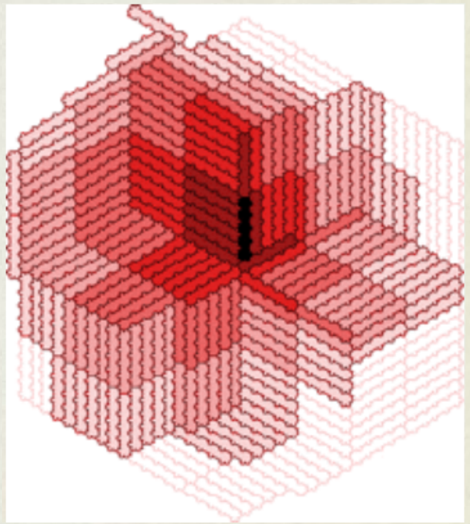
Layer-by-layer growth model
(Zhuravlev 2002, Maleev and Shutov 2011)

The 0-th corona: a tile is defined as the tile itself

The *k*-th corona: the set of tiles **sharing a boundary point** with the ($k$–1)-th corona

1,2 & 3-coronas from a square





https://www.ics.uci.edu/~eppstein/junkyard/heesch/

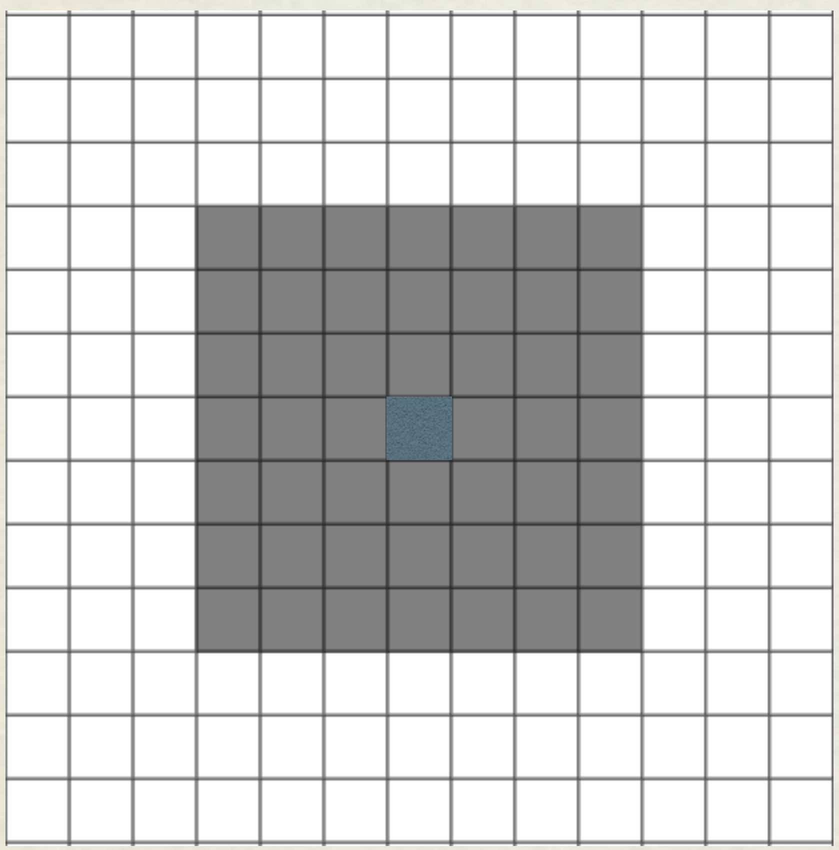
# CORONA & EDGE CORONA

## We can change its adjacency.

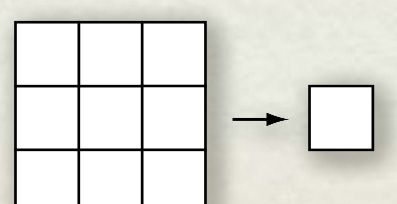
The 0-th **edge corona**: a tile is defined as the tile itself

The $k$-th **edge corona**: the set of tiles sharing a **edge** with the $(k-1)$-th **edge corona**

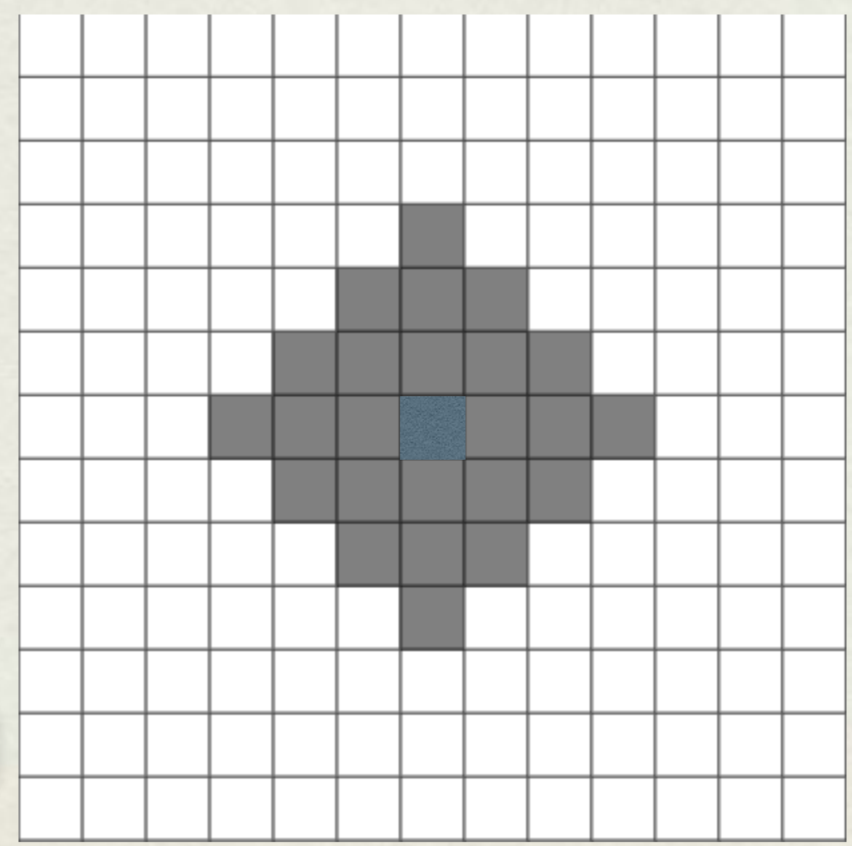
### 3-corona from a square



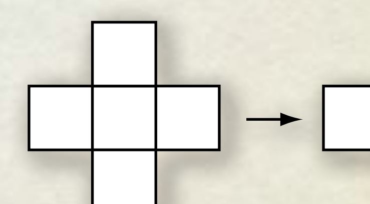
Moore



### 3-edge corona from a square

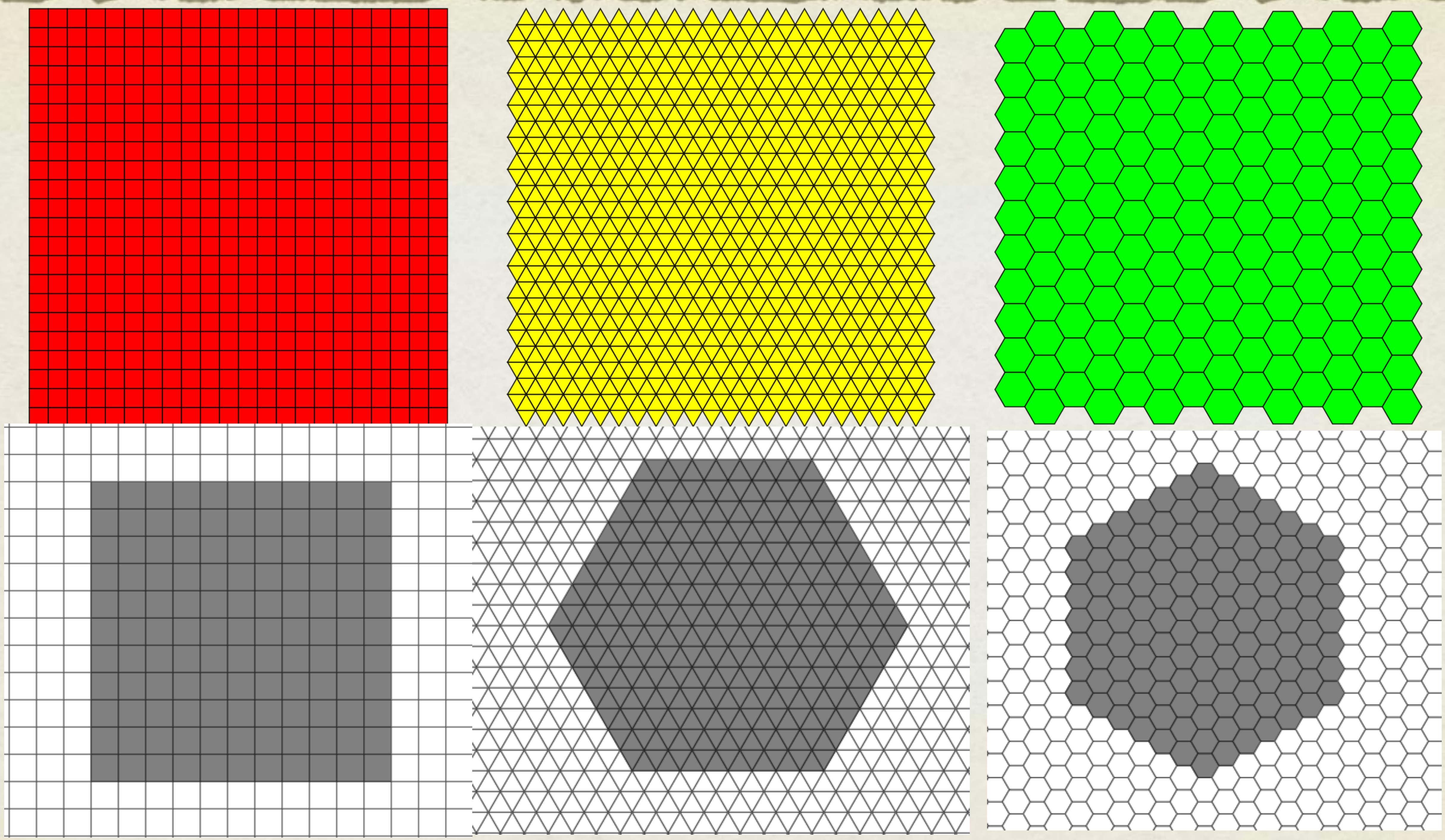


Von Neumann



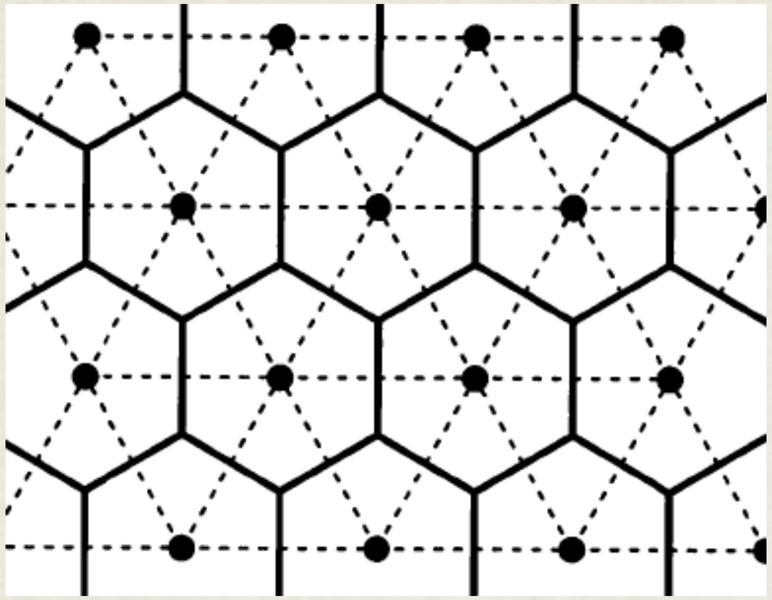
A corona-sequence represents the `light cone' of a cellular space.

# EXAMPLE: CORONAS OF SIMPLE TILINGS



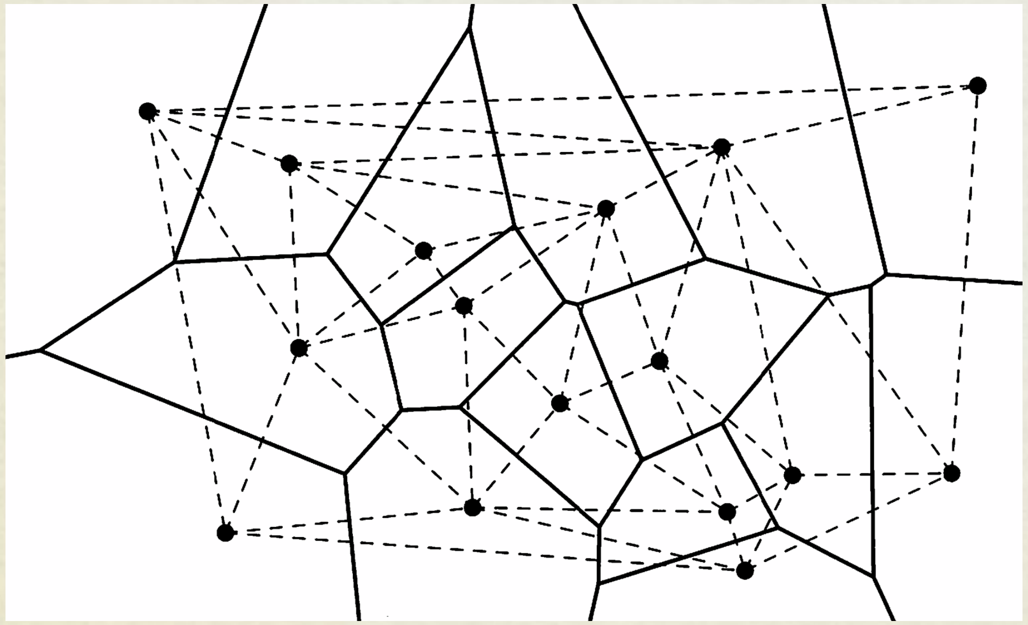
Why not triangle?

# TILING MAY NOT BE LIMITED TO A LATTICE



We can think coronas on any Volonoi cells associated with a (even randomly arranged) point set.

(uniformly locally finite)



Assume:

- the point set has infinite elements
- it fills the whole space

We can think about its corona.

# NEIGHBORHOOD MAY NOT BE LIMITED TO POINT OR EDGE ADJACENT

An adjacency is a reflexive symmetric binary relation:

a) For any tiles $T_i$ and $T_j$, there exist $n_1, \cdots, n_k$ such that
$i = n_1, j = n_k$ and $T_{n_h} \sim T_{n_{h+1}}$ for $h = 1, \cdots, k-1$.

b) There exists a positive integer $M_1$ such that if $T_i \cap T_j \neq \emptyset$
then $k$ in a) can be chosen to be not greater than $M_1$.

c) There exists a positive real $R$ such that $T_i \sim T_j$ implies

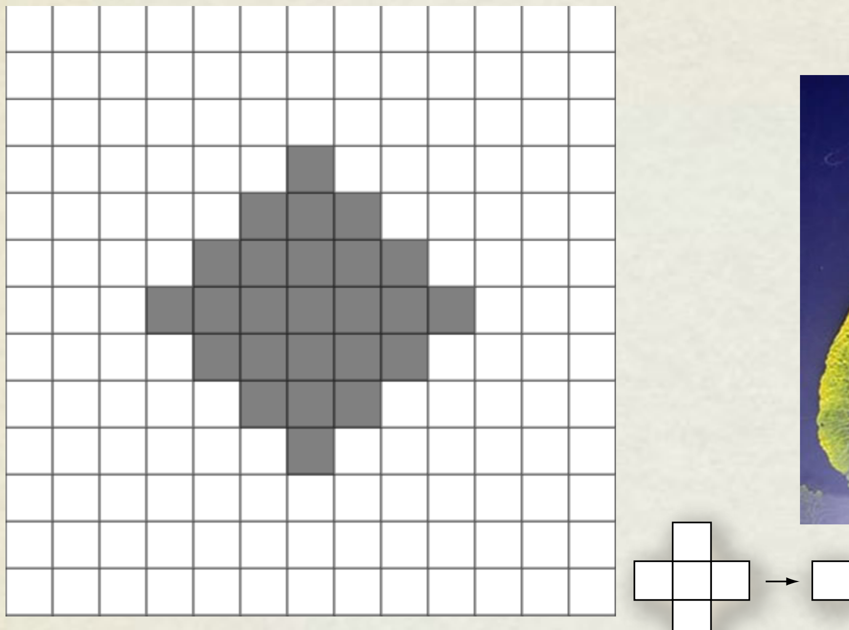
$$d_H(T_i, T_j) \leq R.$$

$d_H$ : Hausdorff distance

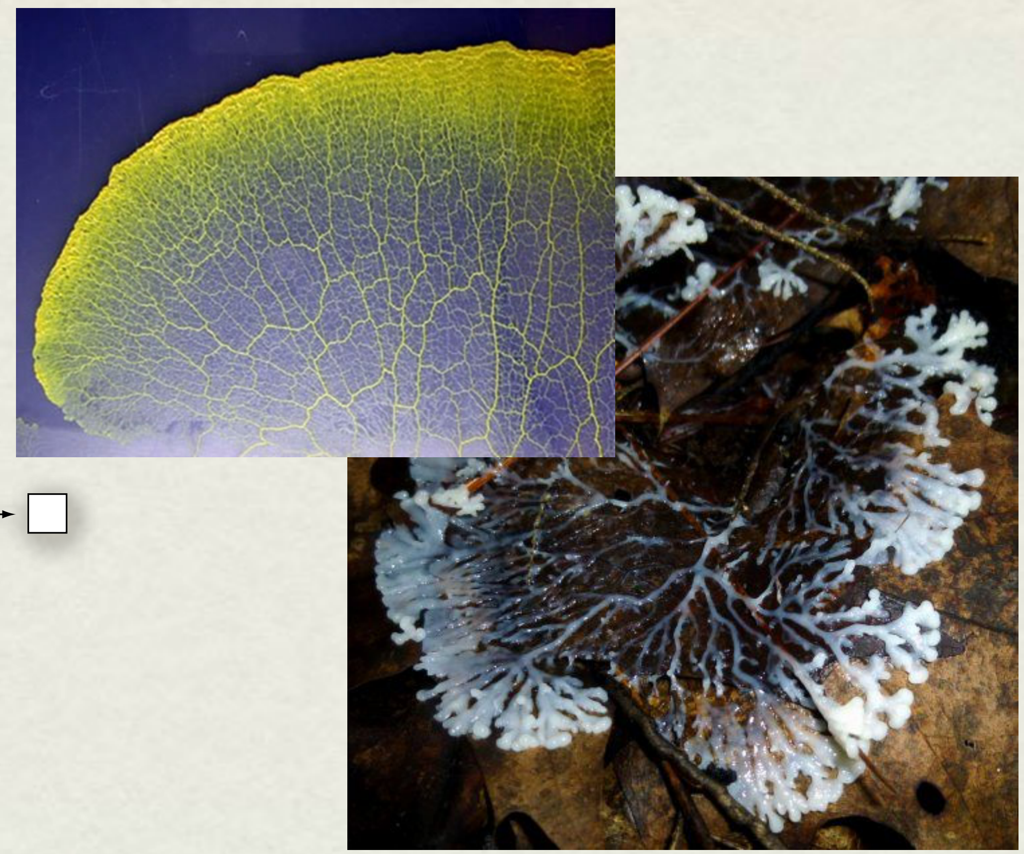
$T_i$ is in the neighborhood of $T_j$ if $T_i \sim T_j$.

We **might** capture the **difference of the local bond energy** of atoms by employing a **proper adjacency relation**.

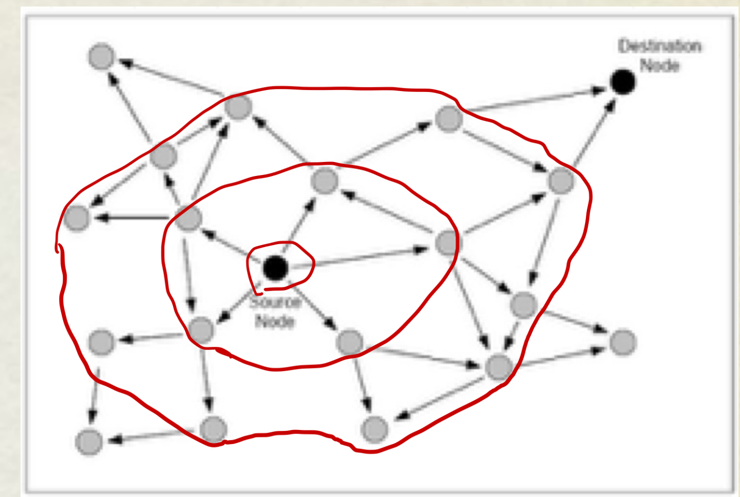
# CORONA AS A WAVEFRONT OF SIGNAL PROPAGATION



The ‘light cone’ of a cellular space.



The shape of the wave front of amoeba growth.




The wavefront of a shape formed by a graph flooding algorithm.

cf. first passage percolation with probability 1

The model is so elementary that I thought there should be many related studies.

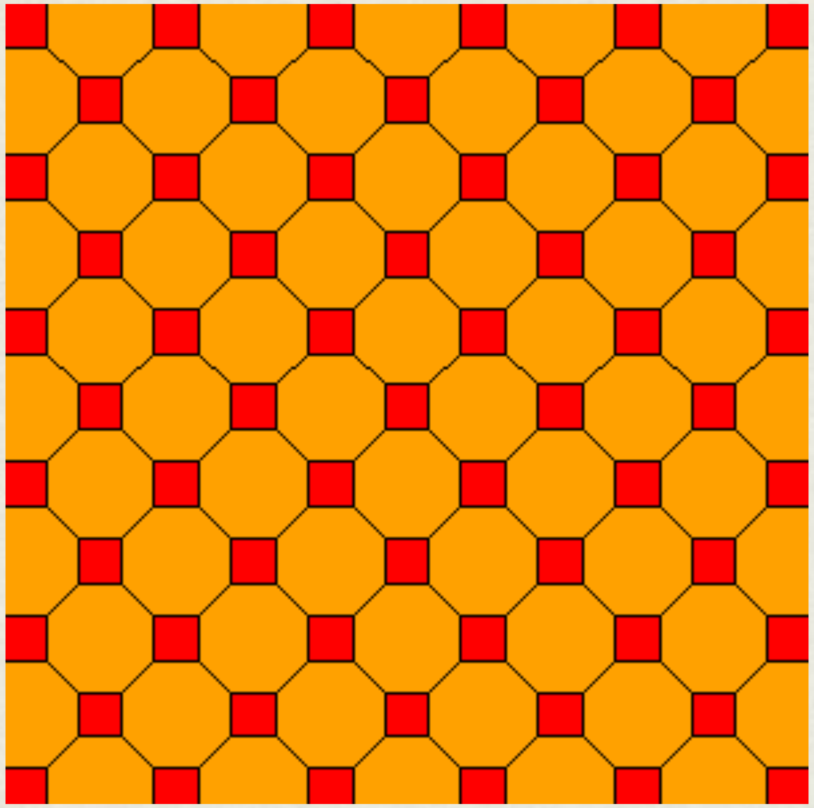
Particularly the periodic cases, it should have already been studied,  maybe 100 years ago…

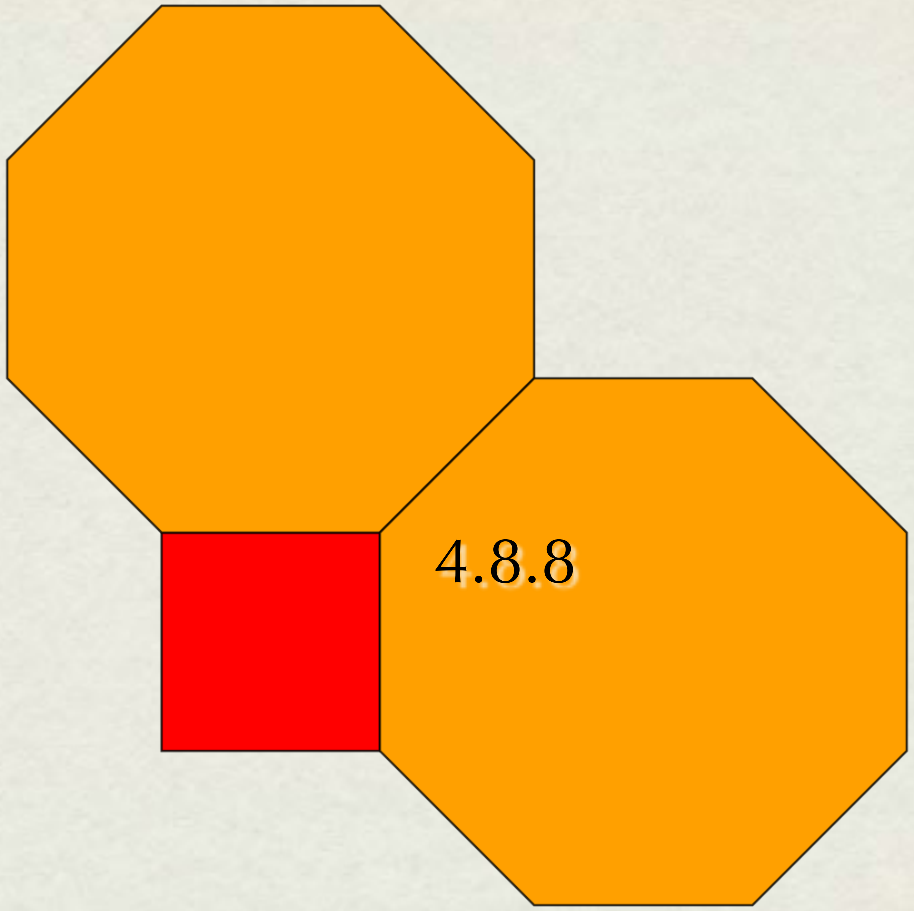
But the related studies are quite limited…

Let's start from

# REGULAR TILINGS

## 1-uniform

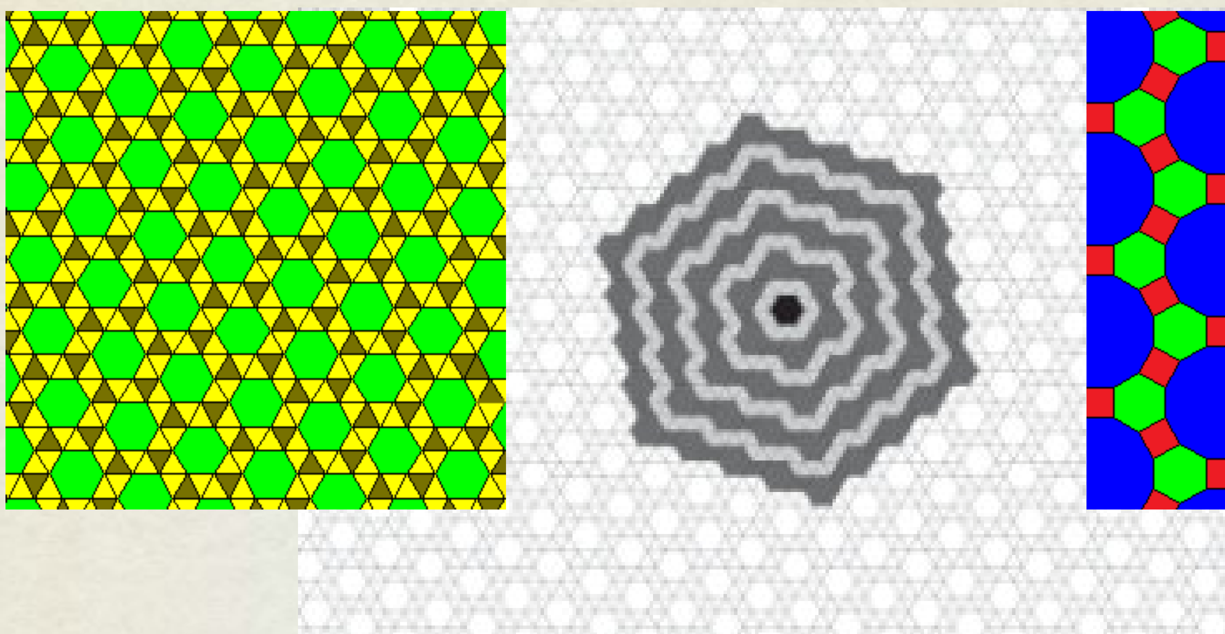




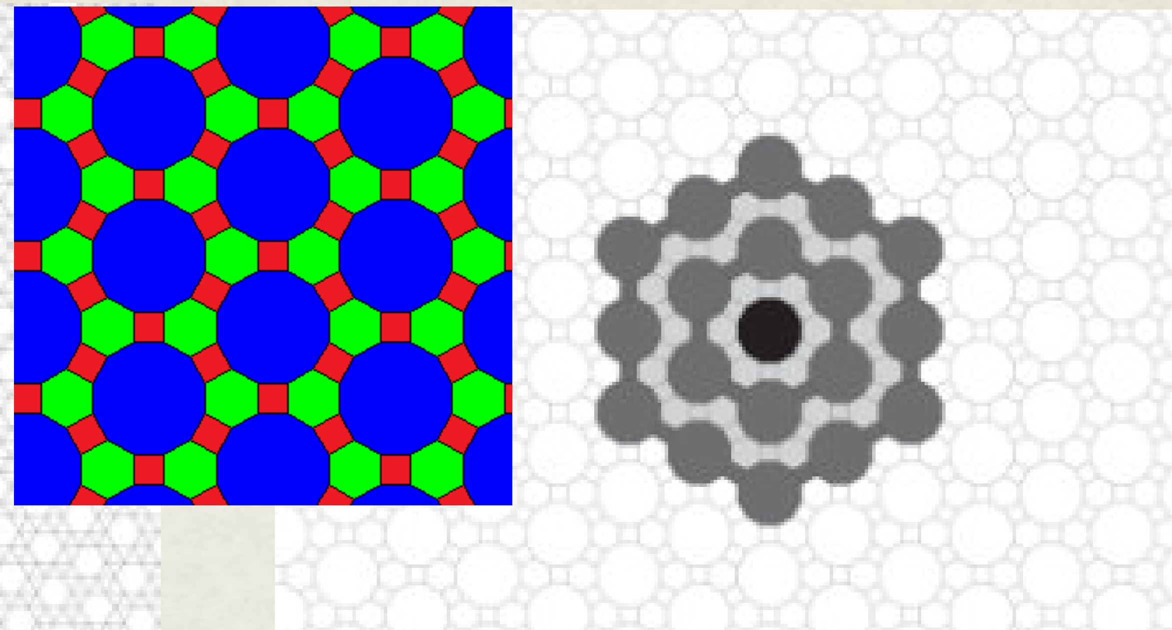

https://en.wikipedia.org/wiki/Euclidean_tilings_by_convex_regular_polygons

# CORONA OF ALCHMEDAN TILINGS 1

33336



46C



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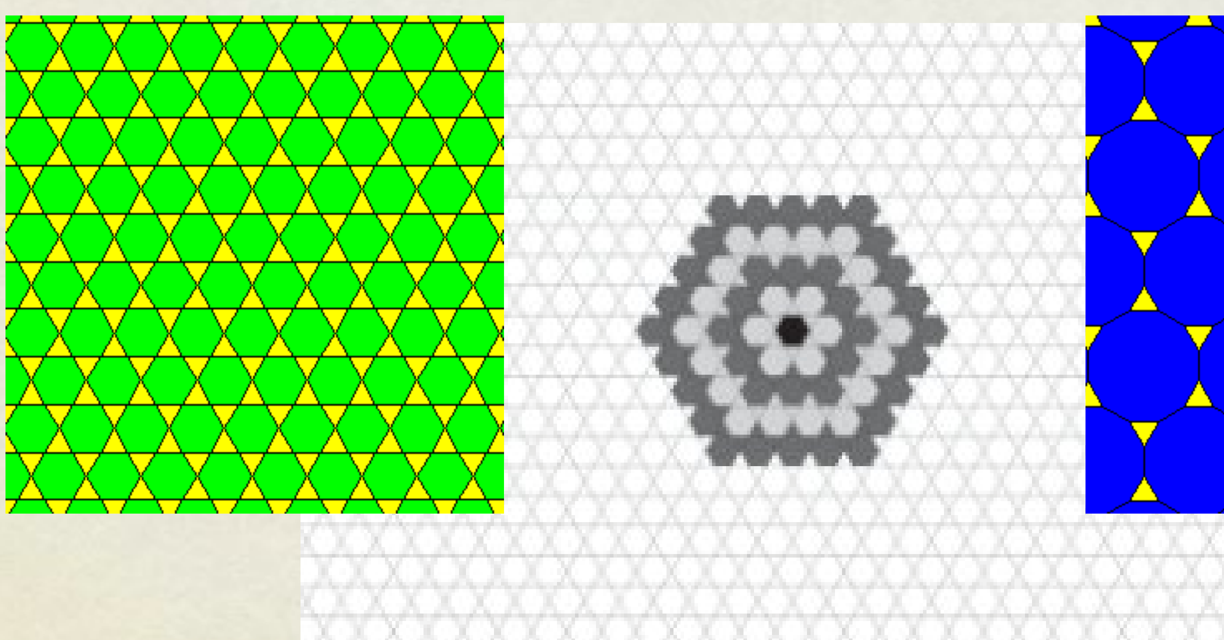
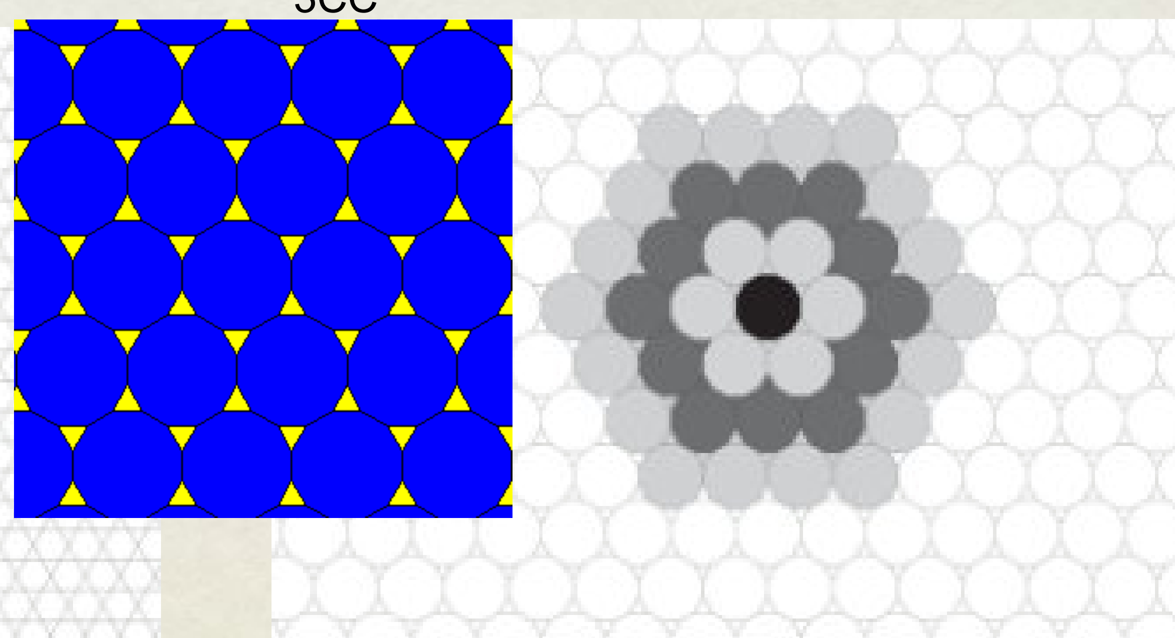
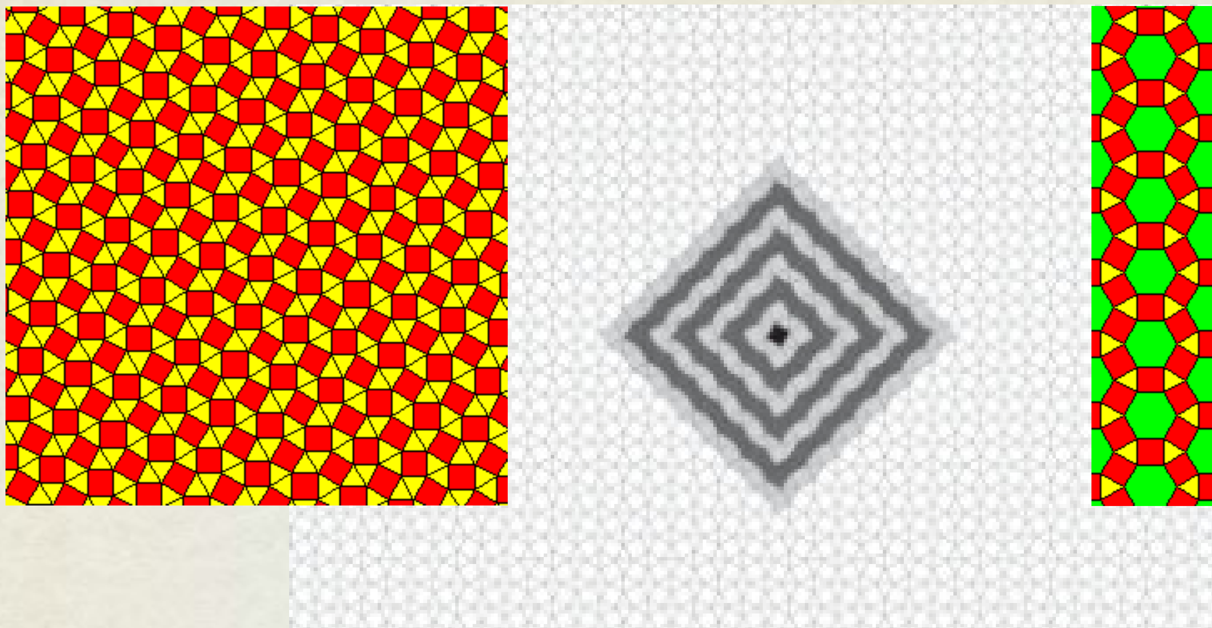


3CC



# CORONA OF ALCHMEDAN TILINGS 2

33434



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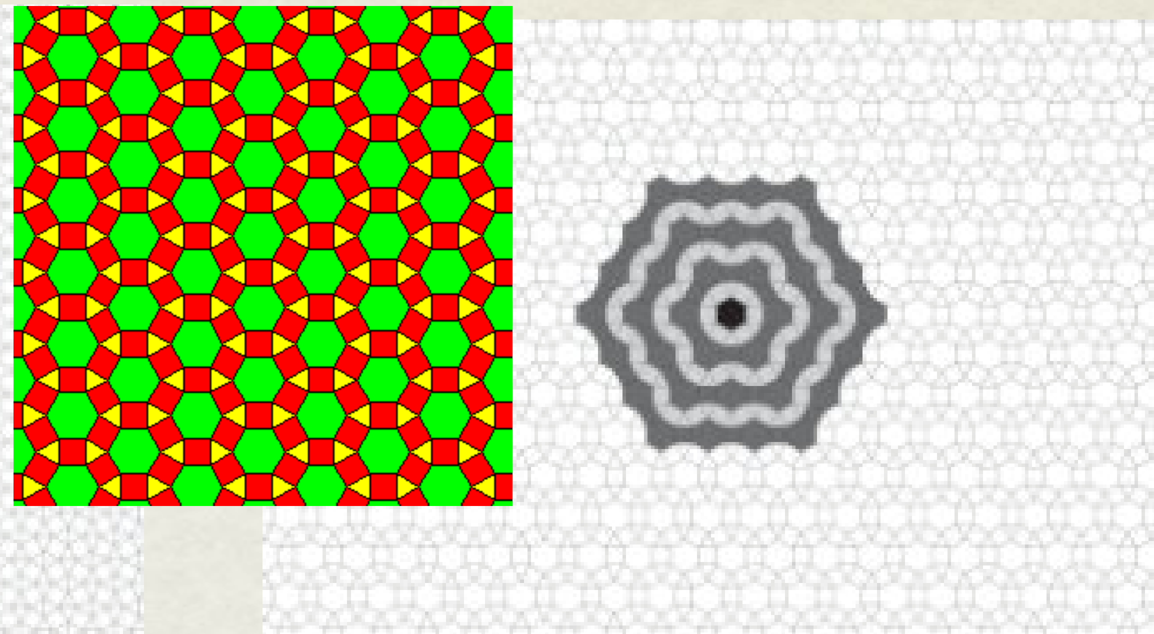

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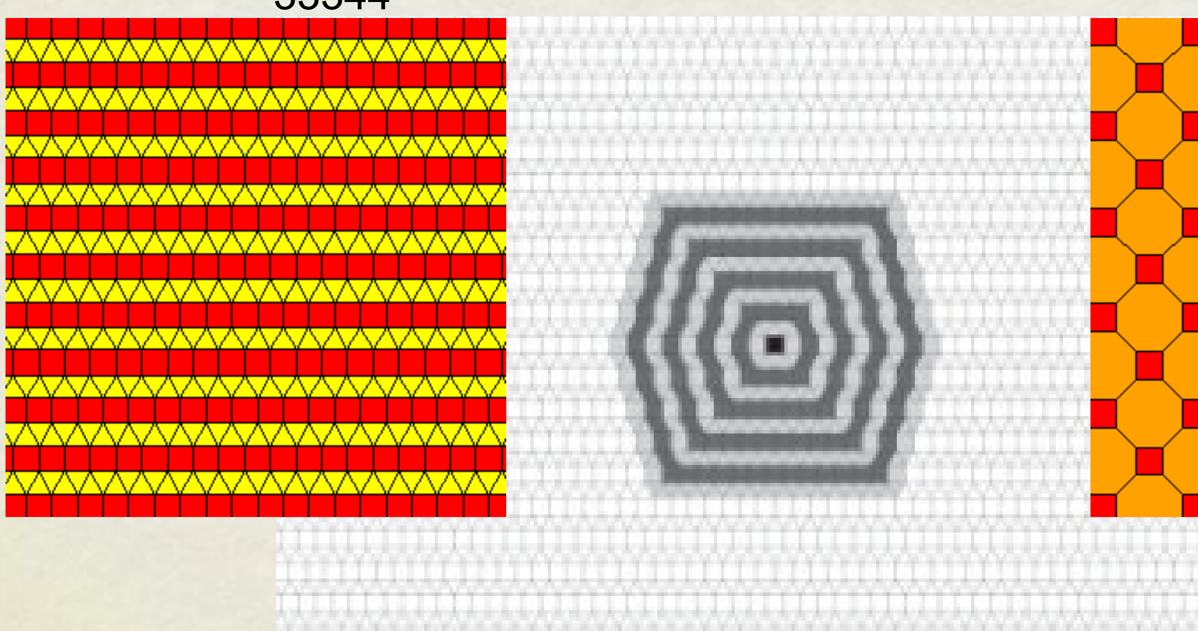

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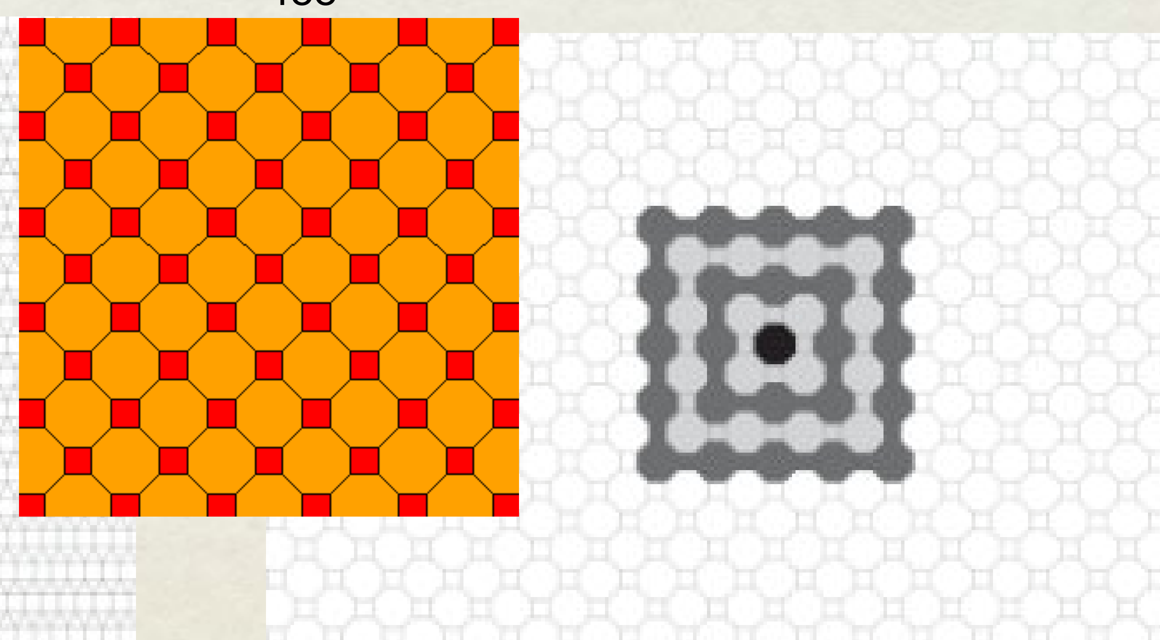
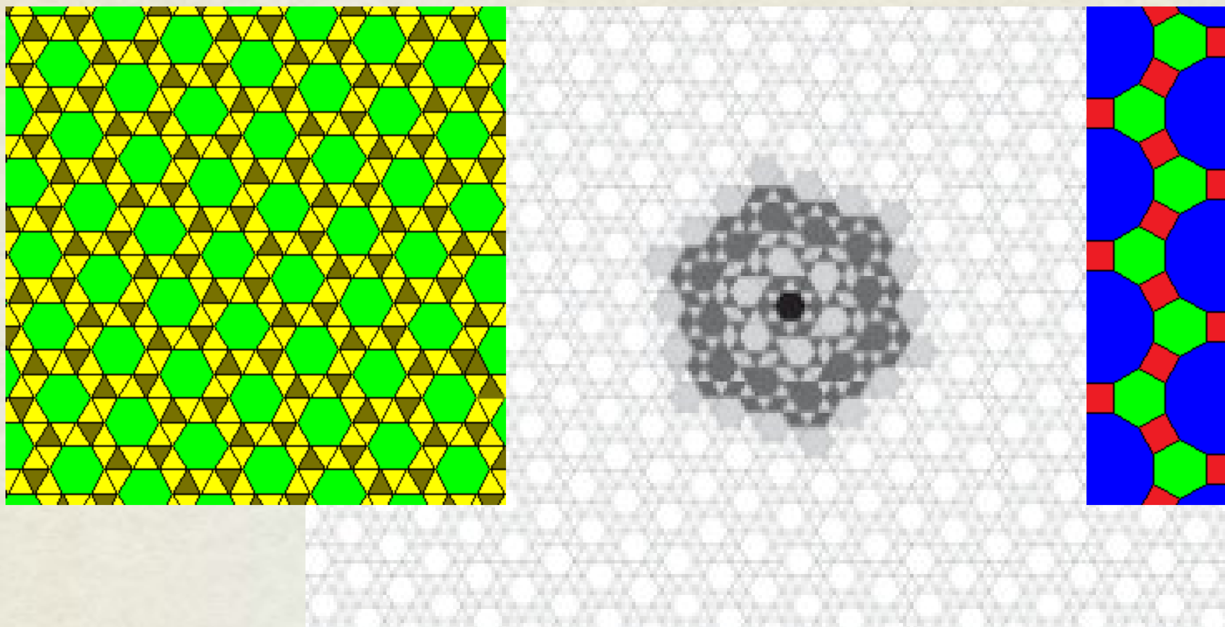


# EDGE CORONA OF ALCHMEDAN TILINGS 1

33336



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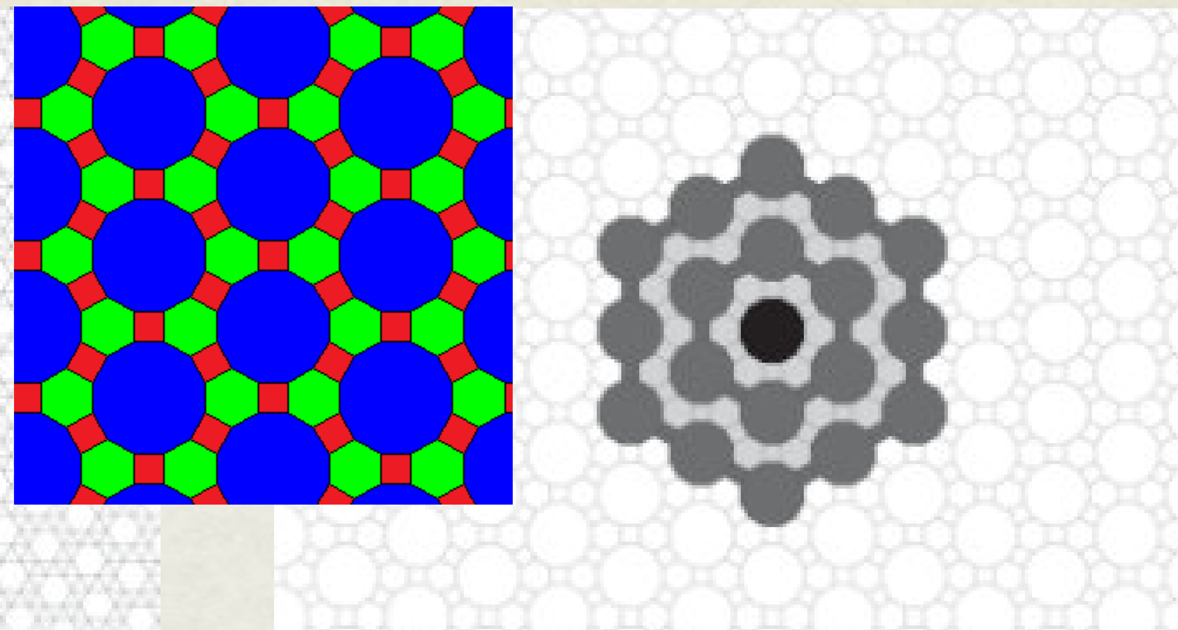

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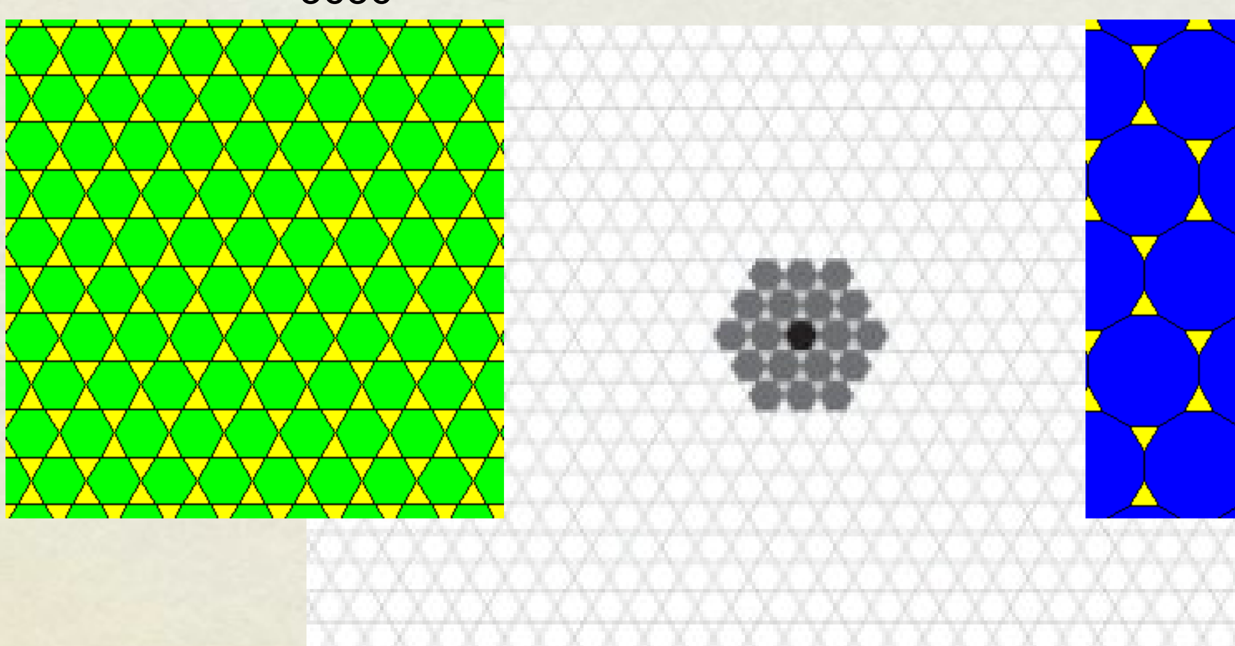

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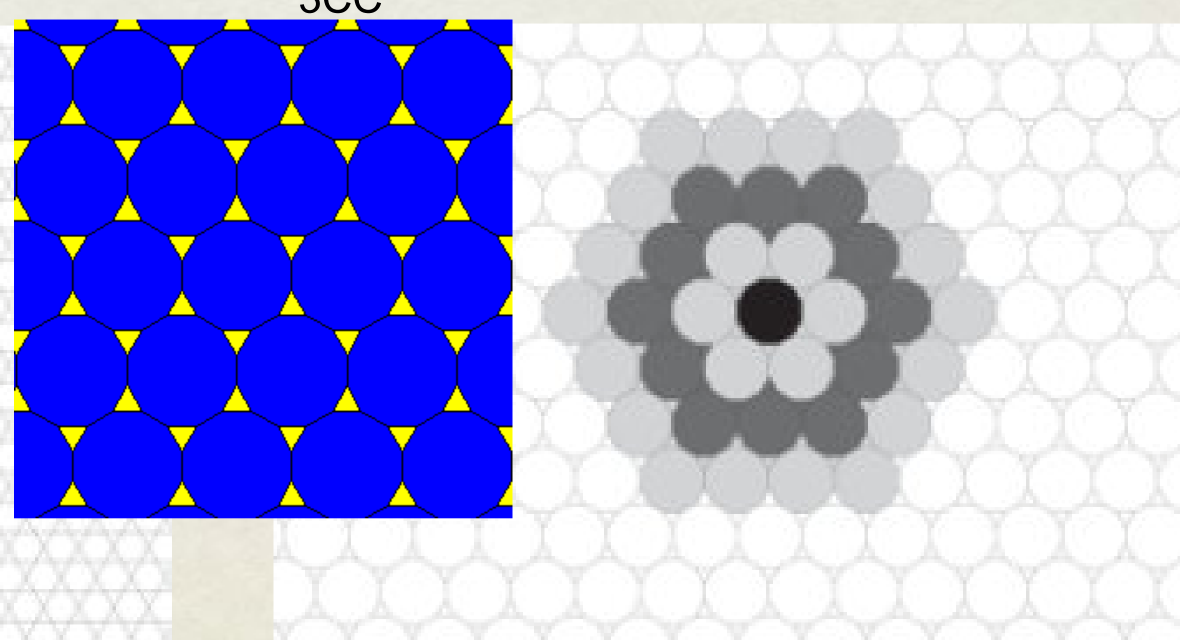

# EDGE CORONA OF ALCHMEDAN TILINGS 2

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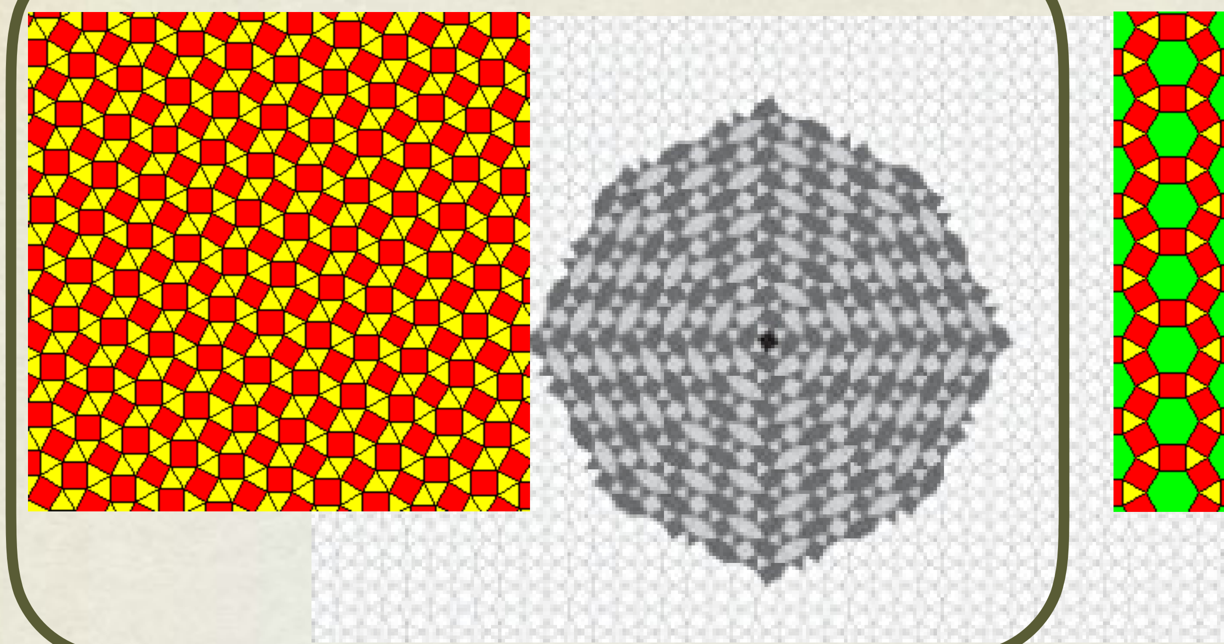

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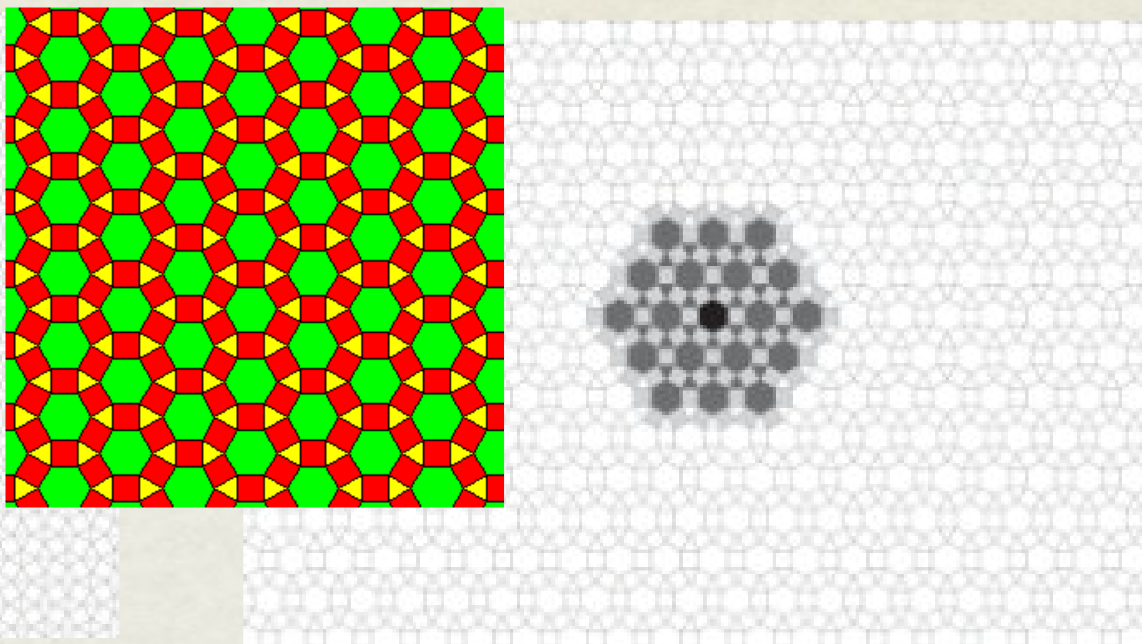

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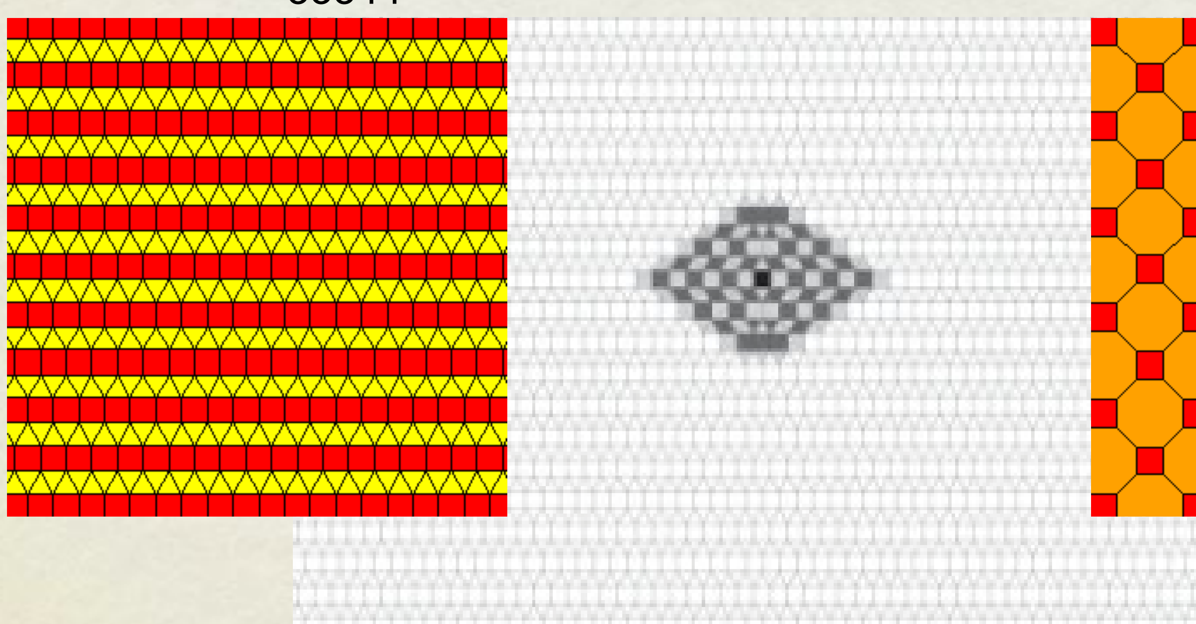

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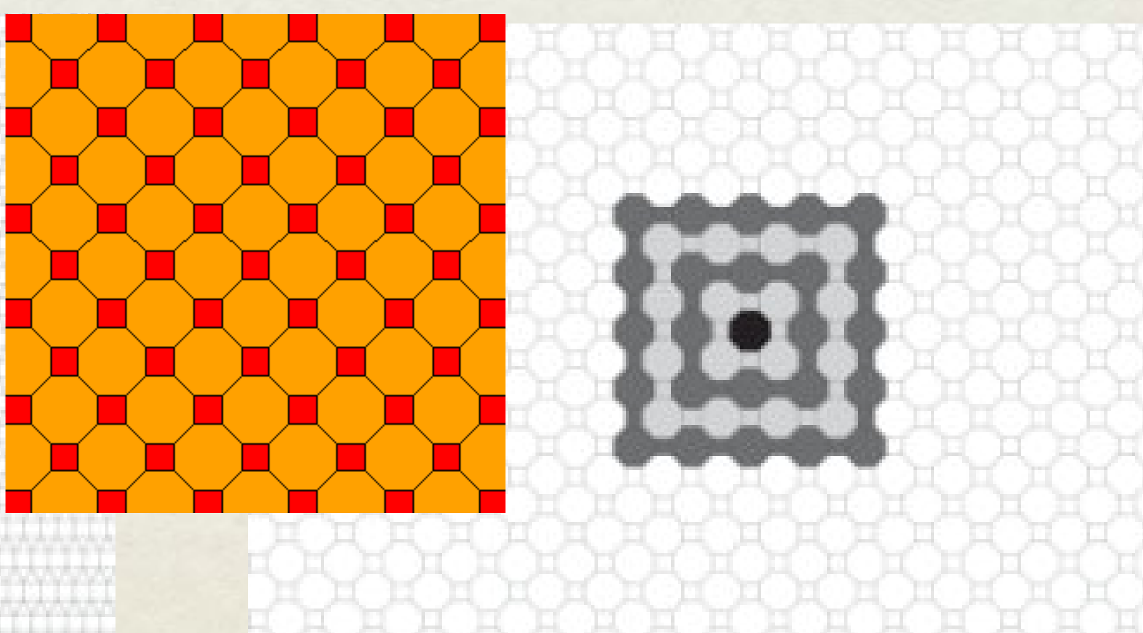
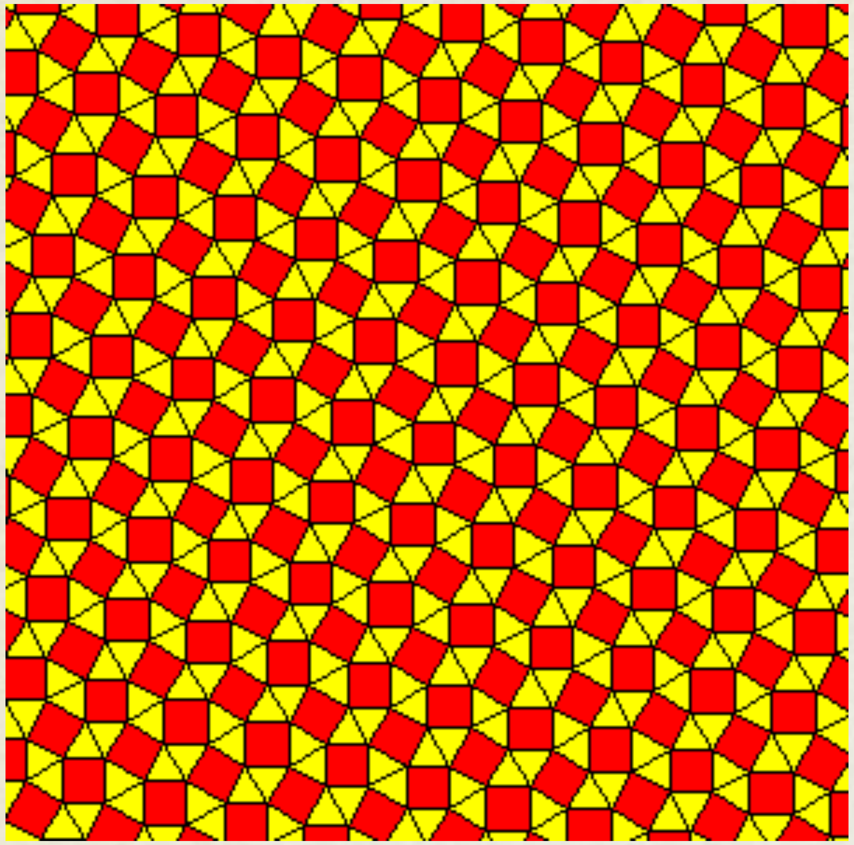
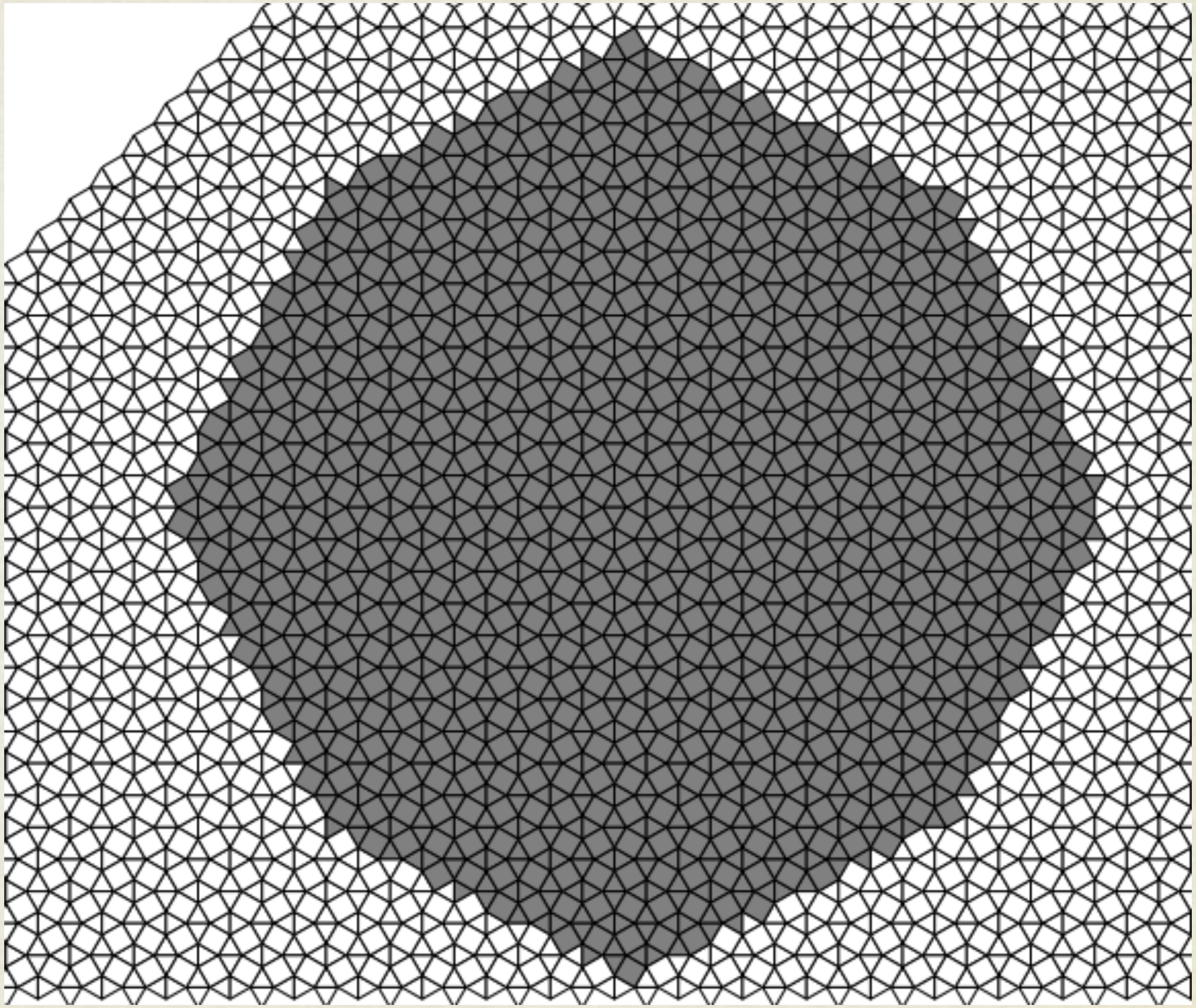
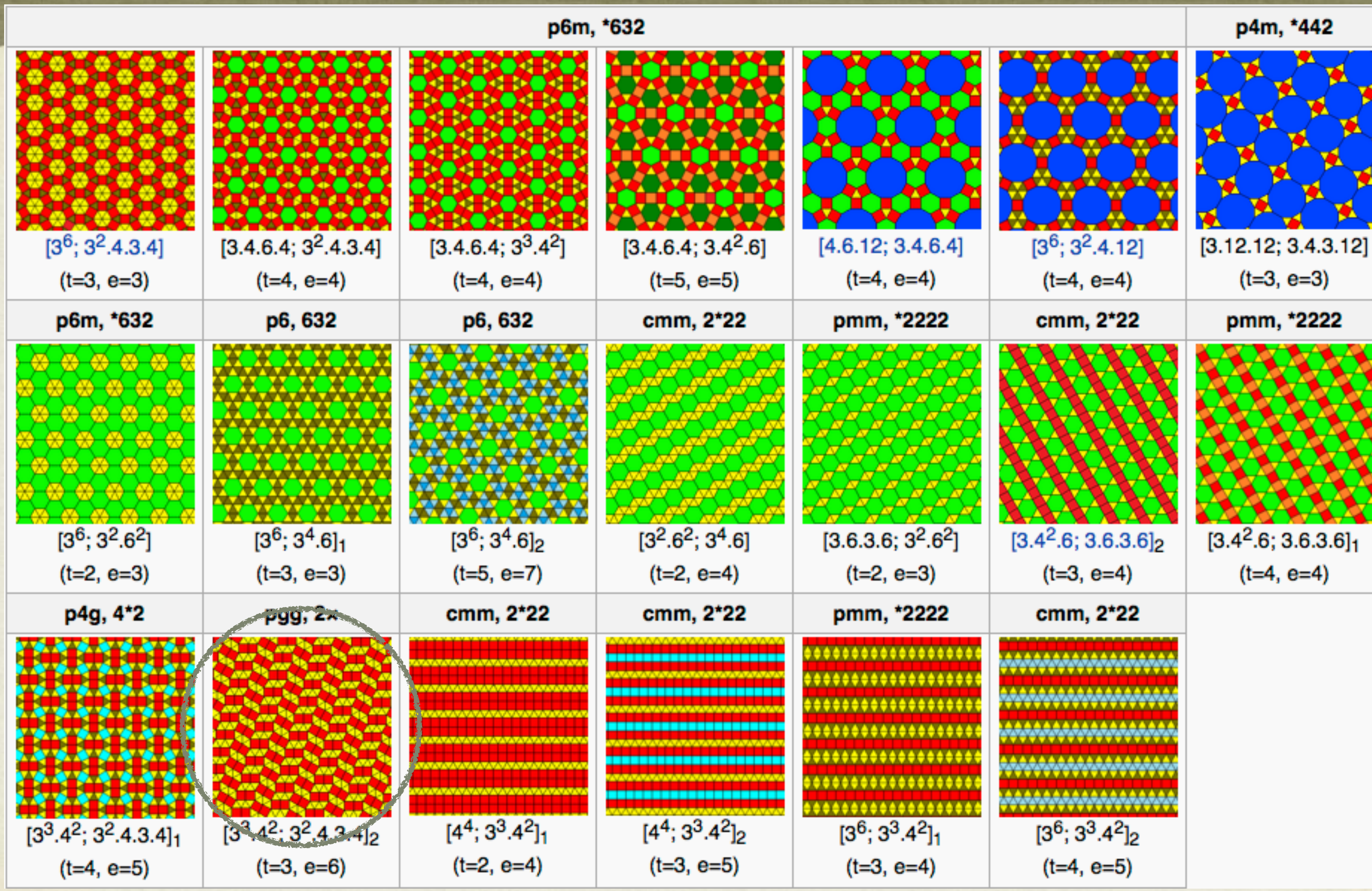


# EDGE CORONA OF 33434



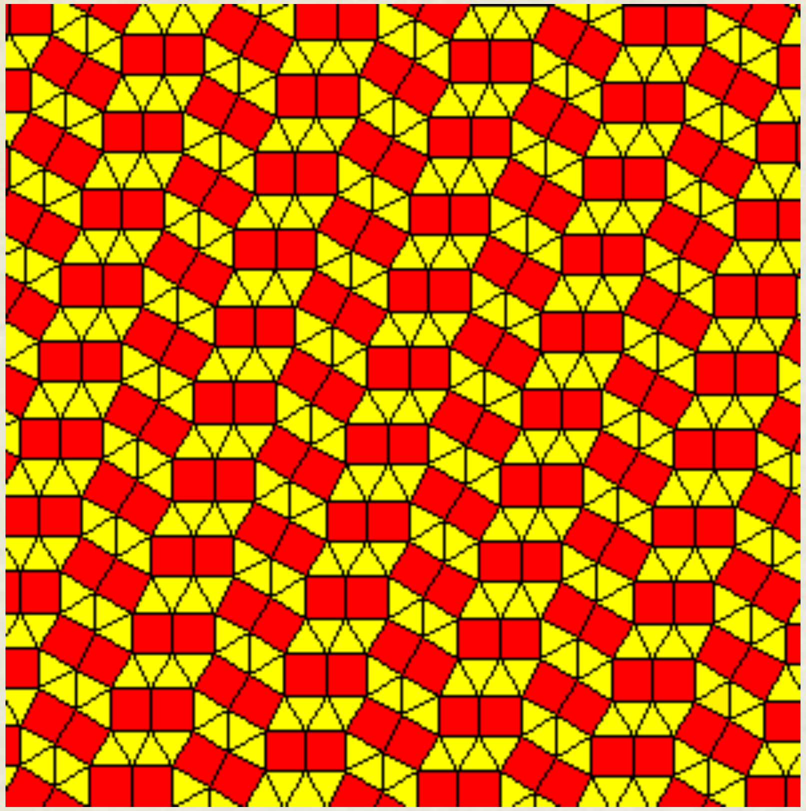


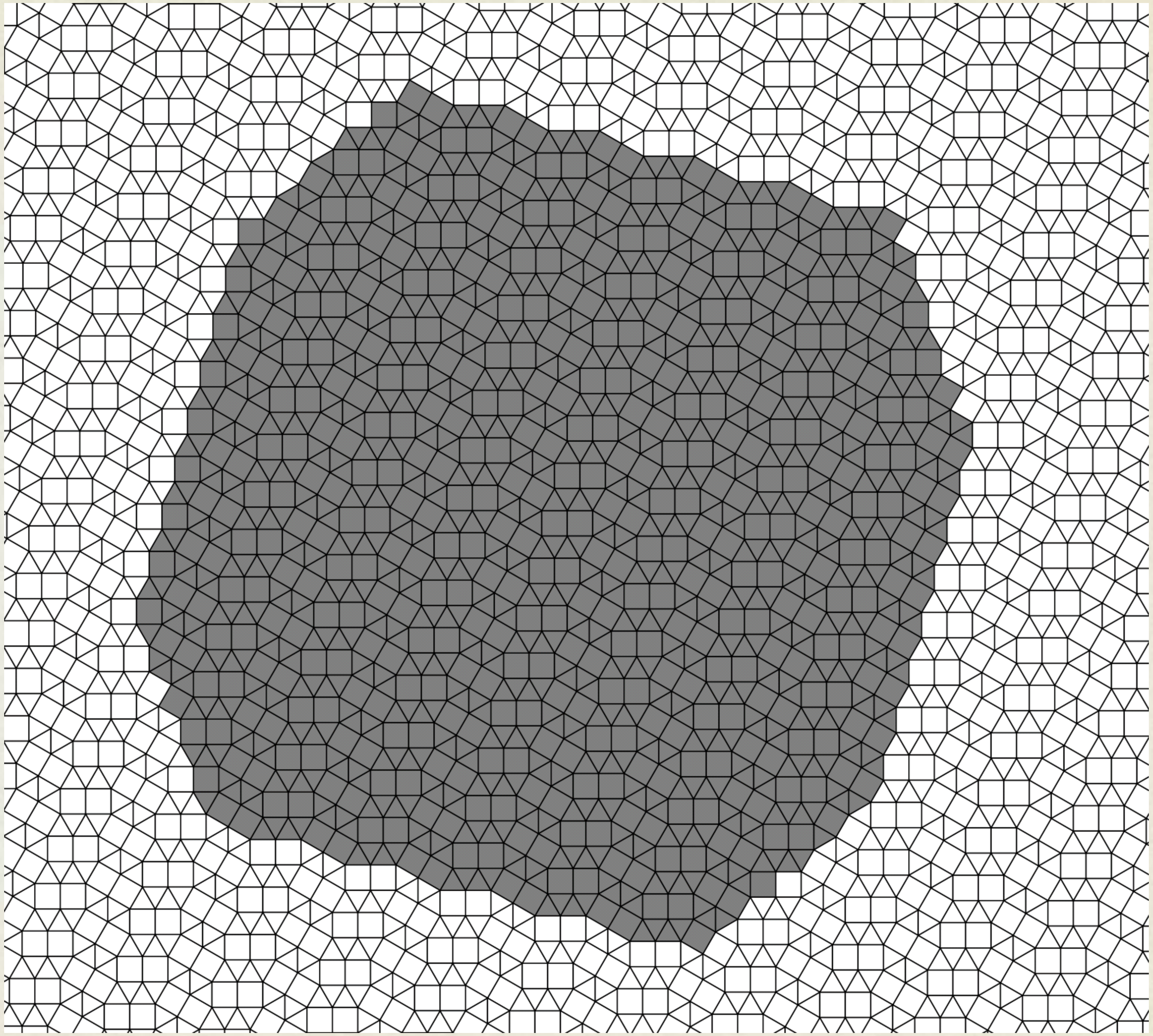
# 2-UNIFORM TILINGS



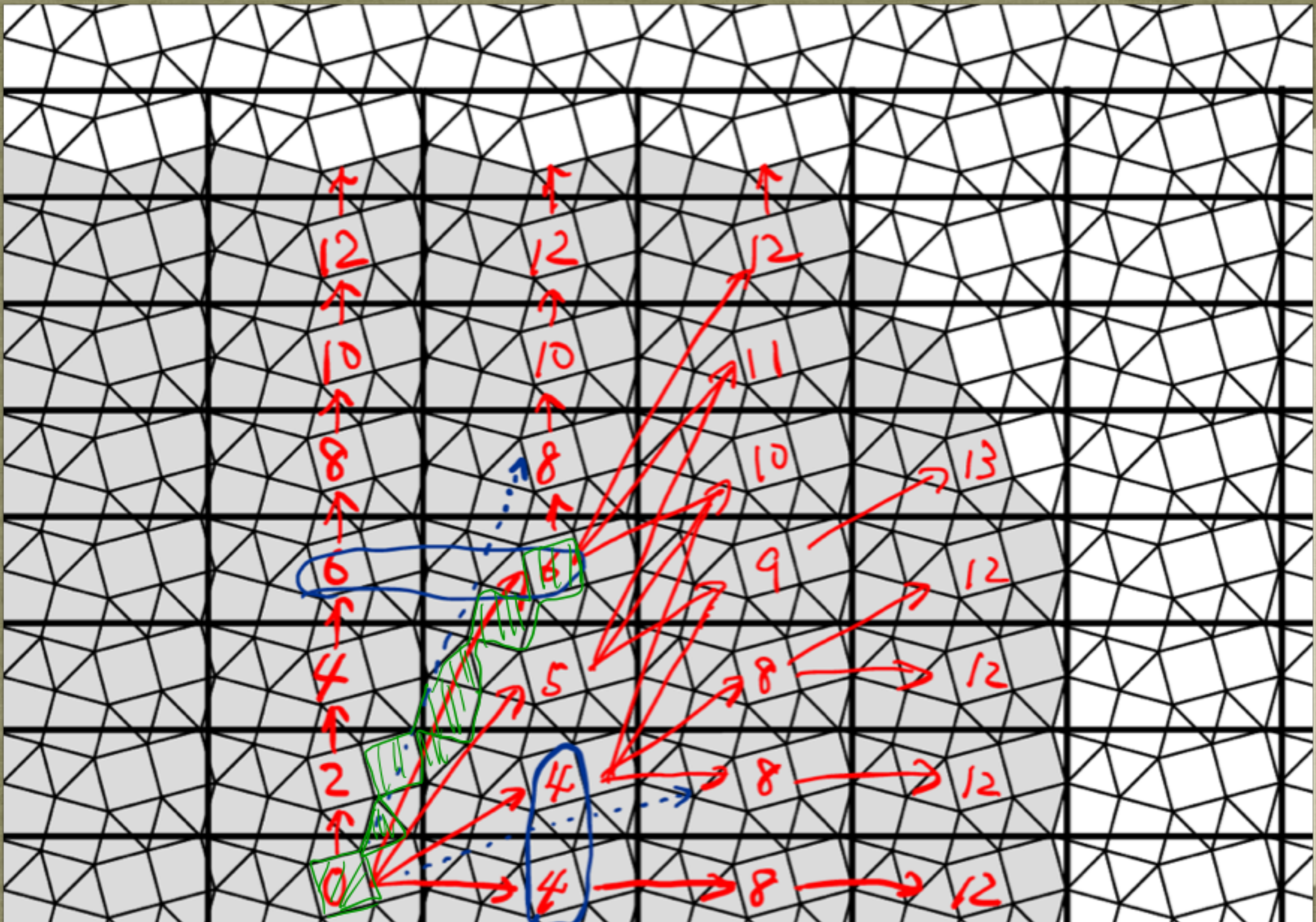

https://en.wikipedia.org/wiki/Euclidean_tilings_by_convex_regular_polygons

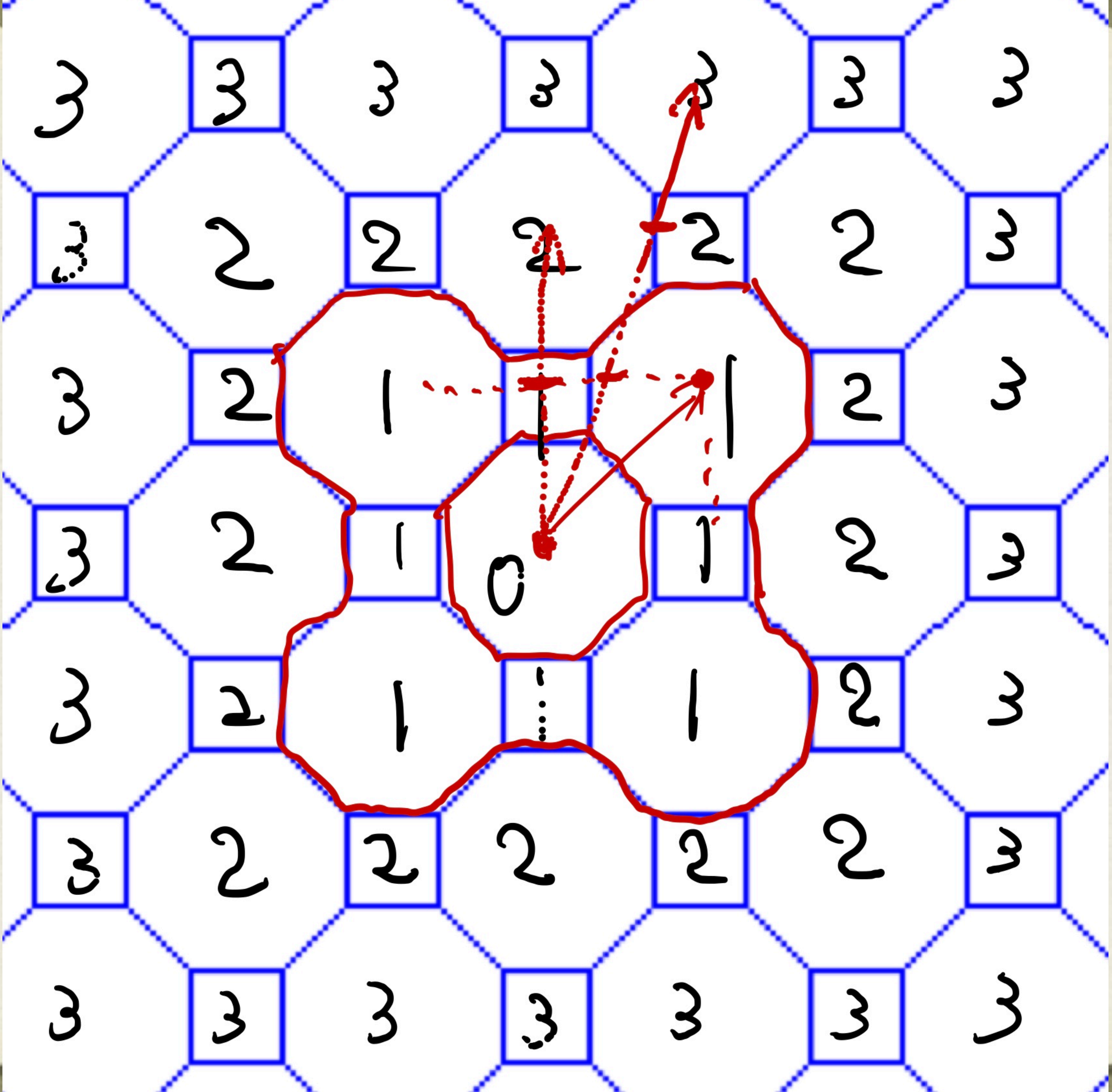
# CORONA OF 33344,33434





## A set of irreducible maximal speed vectors seems to determine the limit shape.

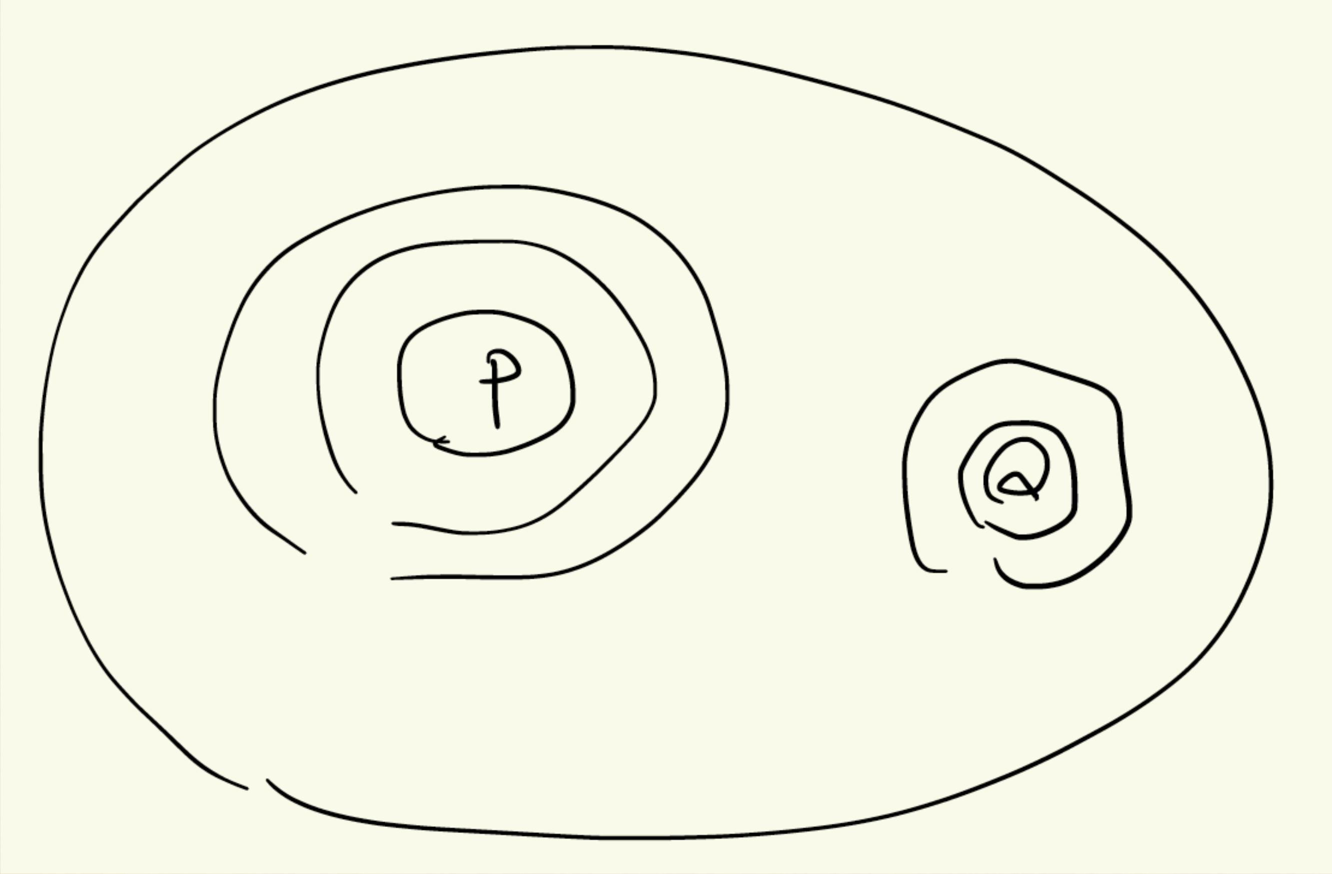




3 3 3 3 3 3 3
3 2 2 2 2 2 3
3 2 1 1 2 3
3 2 1 0 1 2 3
3 2 1 1 2 3
3 2 2 2 2 2 3
3 3 3 3 3 3 3

# ABOUT THE EXISTENCE OF THE CORONA LIMIT OF A TILING

If a corona limit from a finite patch exists then
the other finite patches has their corona limits
and they are the same shape.



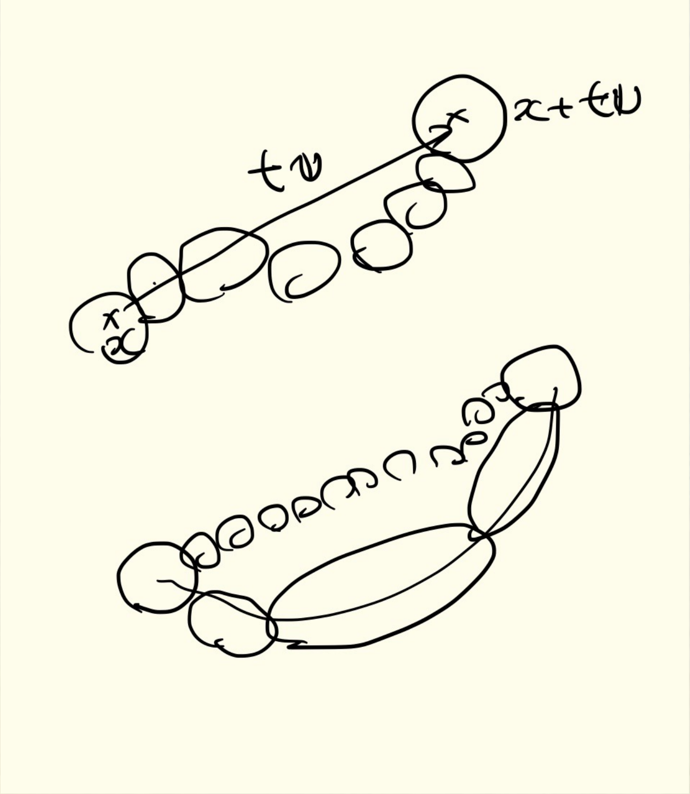

# DIRECTIONAL SPEED

$n(x, y) \in \mathbb{N}$, $\quad n$ is the minimum steps between $x$ and $y$.

$n(x, y) = 0$ if $x$ and $y$ belong to a same tile.

Directional speed:

$$\frac{\|t\mathbf{v}\|}{n(x, x + t\mathbf{v})}$$




$$\overline{d}_1(\mathbf{v}) = \limsup_{t\to\infty} \sup_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x + t\mathbf{v})}$$

$$\overline{d}_2(\mathbf{v}) = \sup_{x\in\mathbb{R}^d} \limsup_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x + t\mathbf{v})}$$

$$\underline{d}_2(\mathbf{v}) = \inf_{x\in\mathbb{R}^d} \liminf_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x + t\mathbf{v})}$$

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# DIRECTIONAL SPEED

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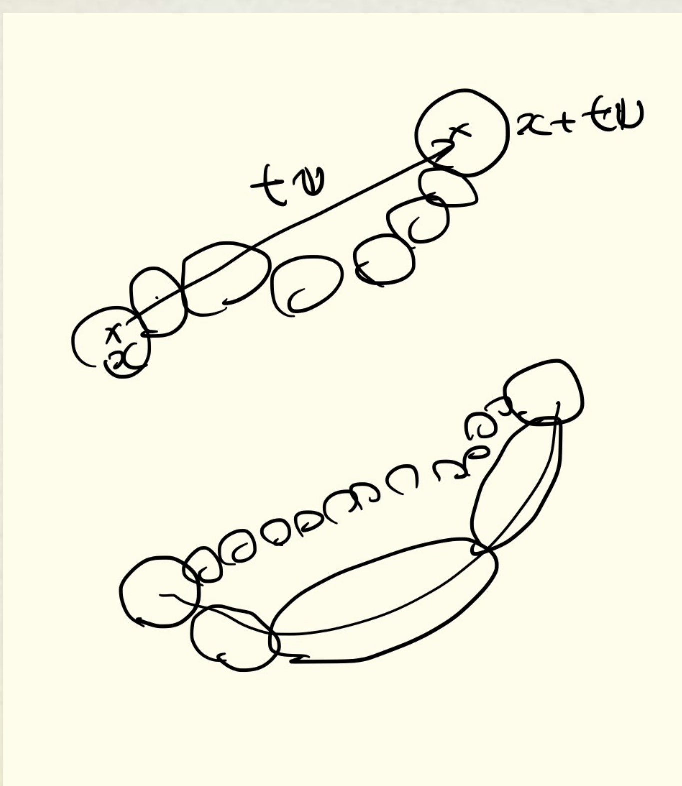
$$\frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$




$$\overline{d}_1(\mathbf{v}) = \limsup_{t\to\infty} \sup_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$\geq$

$$\overline{d}_2(\mathbf{v}) = \sup_{x\in\mathbb{R}^d} \limsup_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$\geq$

$$\underline{d}_2(\mathbf{v}) = \inf_{x\in\mathbb{R}^d} \liminf_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$\geq$

$$\underline{d}_1(\mathbf{v}) = \liminf_{t\to\infty} \inf_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$\underline{d_i}, \overline{d_i}$ are continuous w.r.t. $\mathbf{v}$.

# 1D EXAMPLES

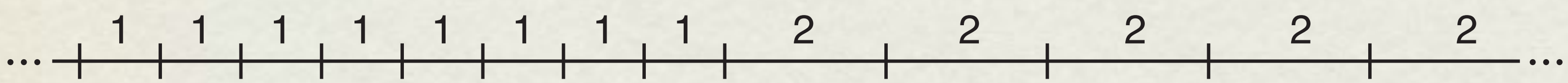
Lemma: $\underline{d}_1(\mathbf{v}) \leq \underline{d}_2(\mathbf{v}) \leq \overline{d}_2(\mathbf{v}) \leq \overline{d}_1(\mathbf{v})$

$$\overline{d}_1(\mathbf{v}) = \limsup_{t\to\infty} \sup_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

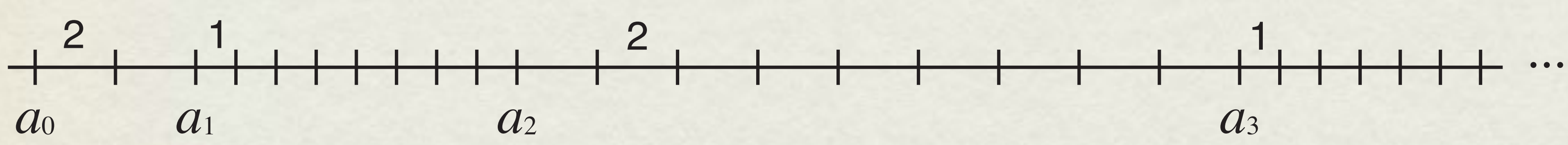
$$\overline{d}_2(\mathbf{v}) = \sup_{x\in\mathbb{R}^d} \limsup_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$$\underline{d}_2(\mathbf{v}) = \inf_{x\in\mathbb{R}^d} \liminf_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$$\underline{d}_1(\mathbf{v}) = \liminf_{t\to\infty} \inf_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$




$$1 = \underline{d}_1(1) < \underline{d}_2(1) = \overline{d}_2(1) = \overline{d}_1(1) = 2$$




$$1 = \underline{d}_2(1) < \overline{d}_2(1) = 2$$

# DIRECTIONAL SPEED

(Akiyama, Caalim, Imai, Kaneko 2017)

$$\overline{d}_1(\mathbf{v}) = \limsup_{t\to\infty} \sup_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$$\overline{d}_2(\mathbf{v}) = \sup_{x\in\mathbb{R}^d} \limsup_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$$\underline{d}_2(\mathbf{v}) = \inf_{x\in\mathbb{R}^d} \liminf_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$$\underline{d}_1(\mathbf{v}) = \liminf_{t\to\infty} \inf_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

Th.1 $\forall \mathbf{v}, \underline{d}_2(\mathbf{v}) = \overline{d}_2(\mathbf{v})$

$\Leftrightarrow$ existence of the corona limit

Corona limit

$$K = \{k\overline{d}_2(\mathbf{v})\mathbf{v} \mid |\mathbf{v}| = 1, k \in [0,1]\}$$

cf. velocity coefficient $K(\theta)$

Th.2 If $\forall \mathbf{v}(\neq 0), \underline{d}_1(\mathbf{v}) = \overline{d}_1(\mathbf{v})$

then the corona limit is convex and point symmetric.

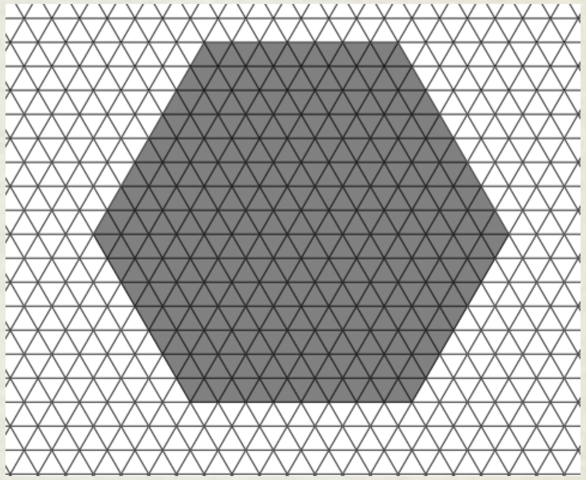
Th.3 If tiling is (lattice) periodic then

the corona limit is a convex and point symmetric polygon.

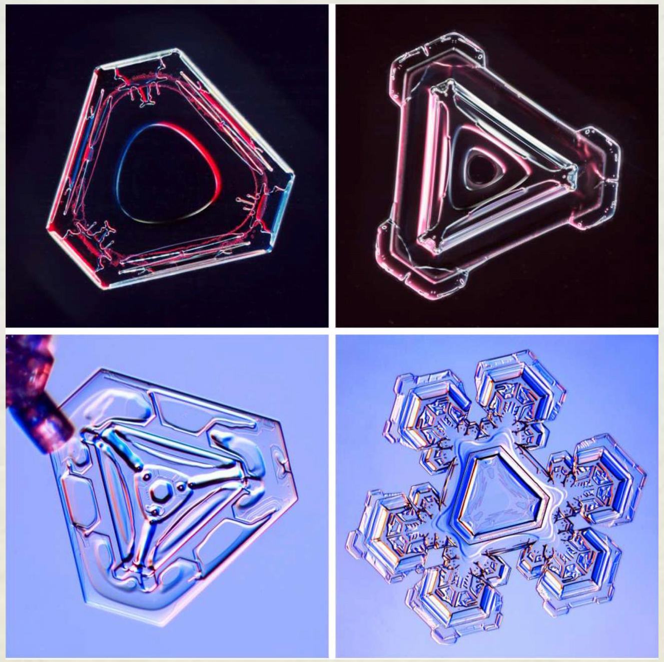
(Zhuravlev 2002, Maleev and Shutov 2011)

The corona limit of any periodic tiling exists and is a point-symmetric convex polygon.

There is no triangular corona limit in the periodic tiling cases.



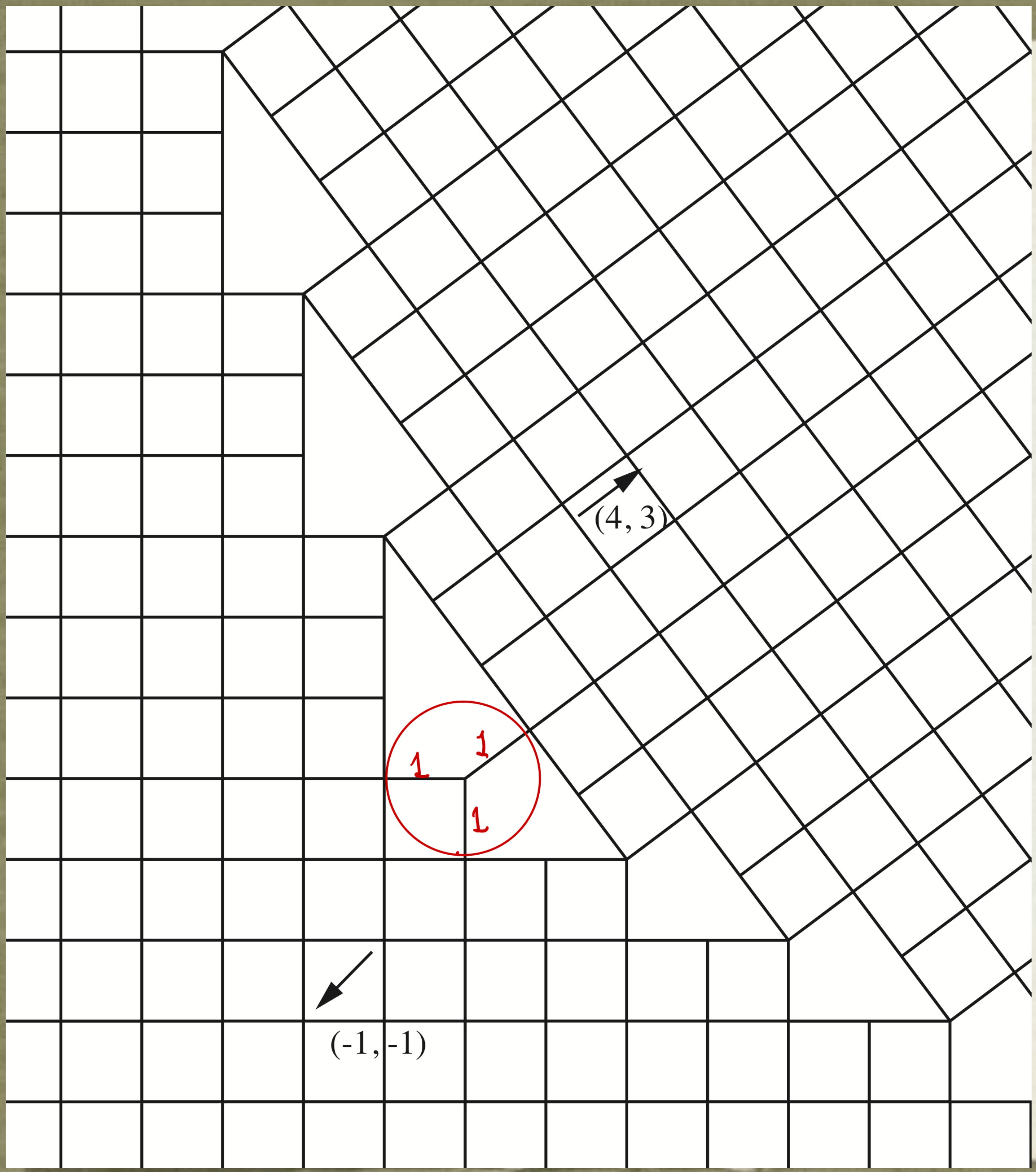
Why not triangle?



Libbrecht and Arnold 2009

But **other factors** such as aerodynamic effect might cause triangular growth...

# 2D EXAMPLE




$$\overline{d}_1(\mathbf{v}) = \limsup_{t\to\infty} \sup_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$$\overline{d}_2(\mathbf{v}) = \sup_{x\in\mathbb{R}^d} \limsup_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$$\underline{d}_2(\mathbf{v}) = \inf_{x\in\mathbb{R}^d} \liminf_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$$\underline{d}_1(\mathbf{v}) = \liminf_{t\to\infty} \inf_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

$$\mathbf{v} = (4, 3)$$

$$1 = \underline{d}_1(\mathbf{v}) = \underline{d}_2(\mathbf{v}) = \overline{d}_2(\mathbf{v}) < \overline{d}_1(\mathbf{v}) = 5/4$$

# CORONA LIMITS OF PERIODIC TILINGS CAN BE EFFECTIVELY COMPUTED

$$K = \{k\overline{d}_2(\mathbf{v})\mathbf{v} \mid |\mathbf{v}| = 1, k \in [0,1]\}$$

$$\overline{d}_1(\mathbf{v}) = \limsup_{t\to\infty} \sup_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

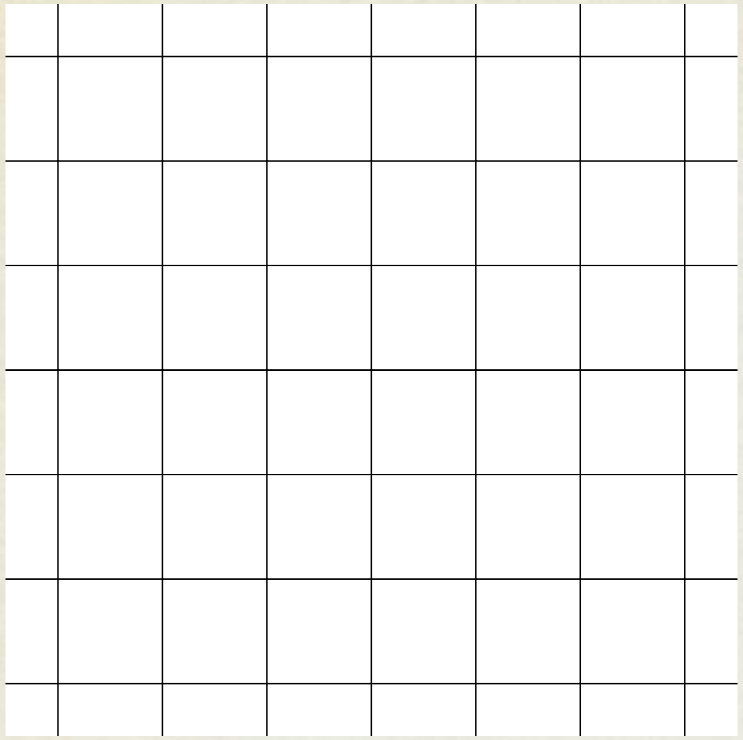
$$\overline{d}_2(\mathbf{v}) = \sup_{x\in\mathbb{R}^d} \limsup_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

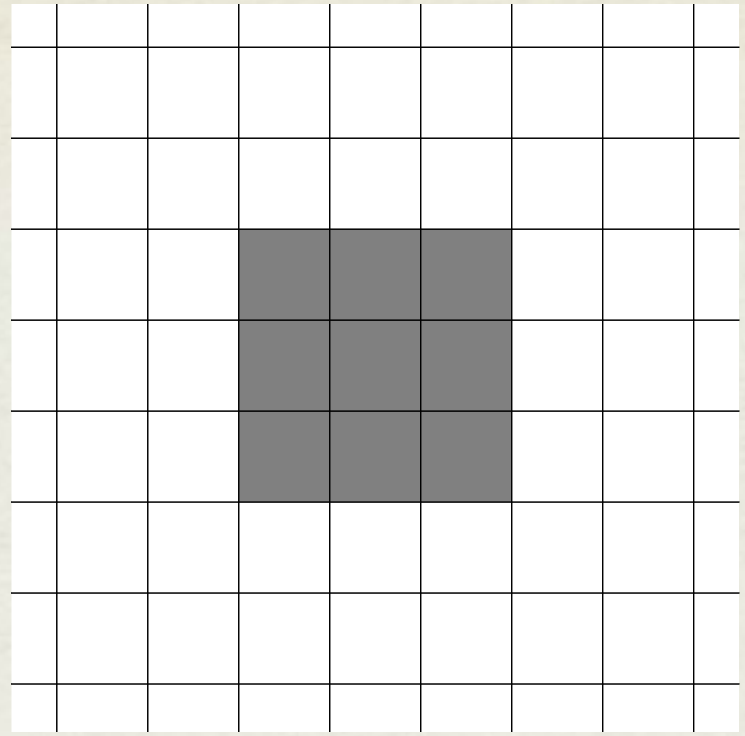
$$\underline{d}_2(\mathbf{v}) = \inf_{x\in\mathbb{R}^d} \liminf_{t\to\infty} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

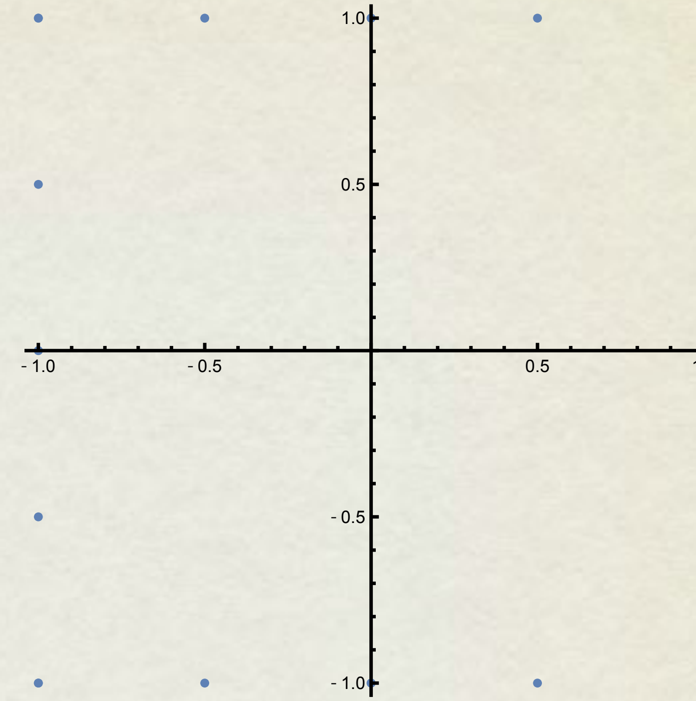
$$\underline{d}_1(\mathbf{v}) = \liminf_{t\to\infty} \inf_{x\in\mathbb{R}^d} \frac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$$

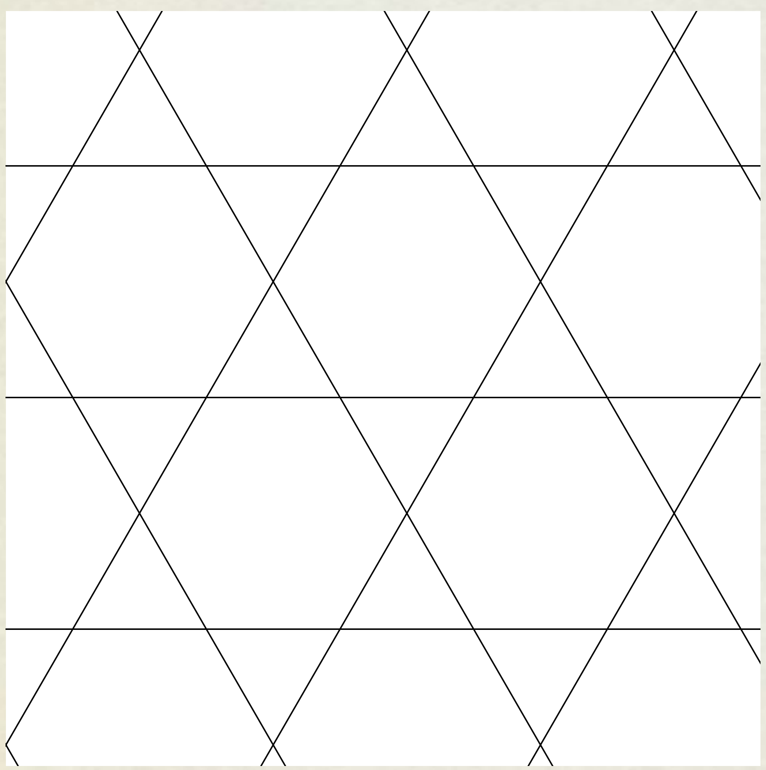
The corona limit $K$ of a periodic tiling can be computed by inspecting $\dfrac{\|t\mathbf{v}\|}{n(x, x+t\mathbf{v})}$ at most $|T|+1$-corona.

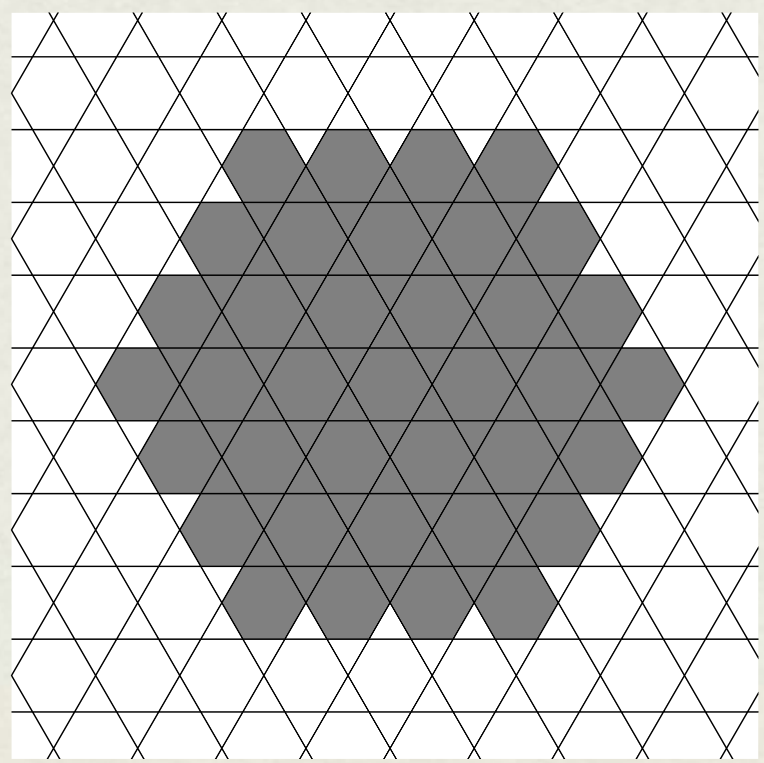
cf. velocity coefficient $K(\theta)$

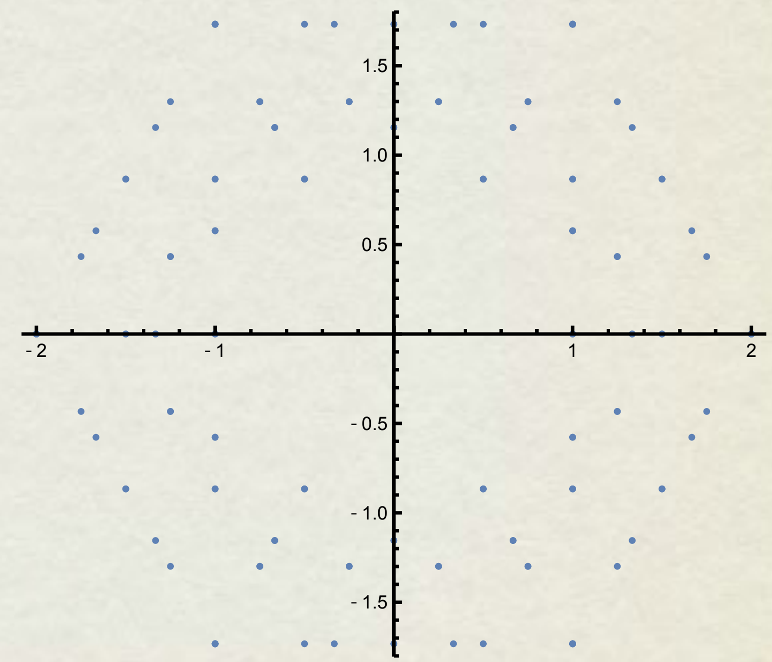


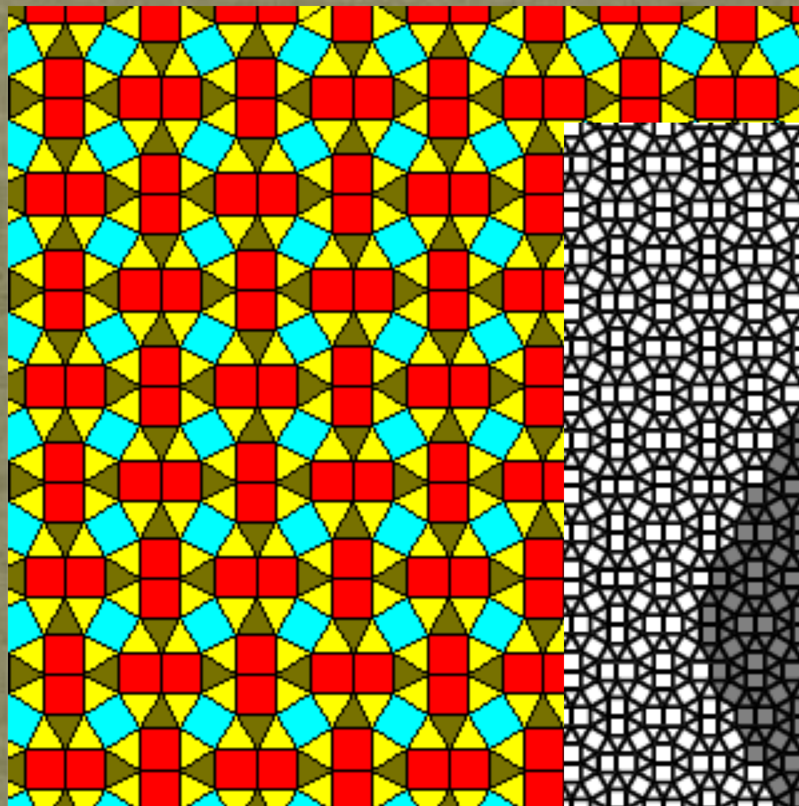



1.0
0.5
- 1.0
- 0.5
0.5
1.0
- 0.5
- 1.0

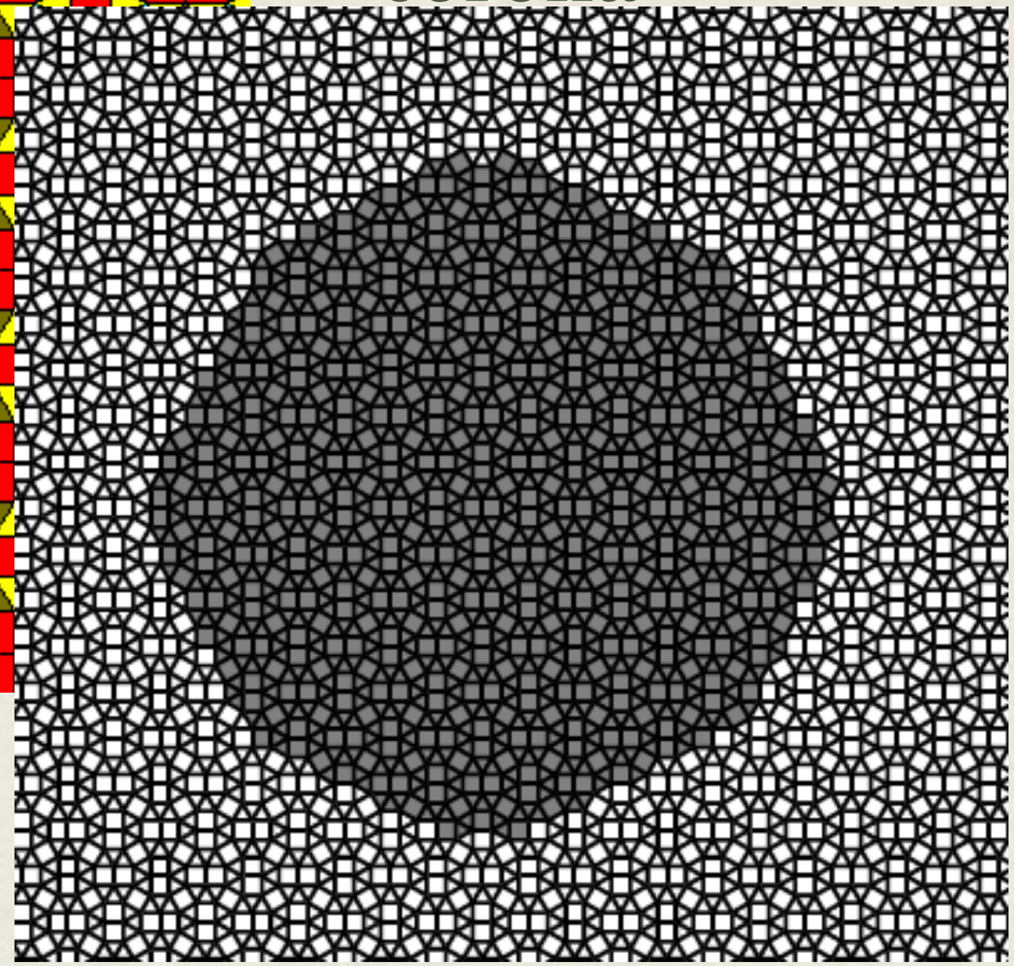




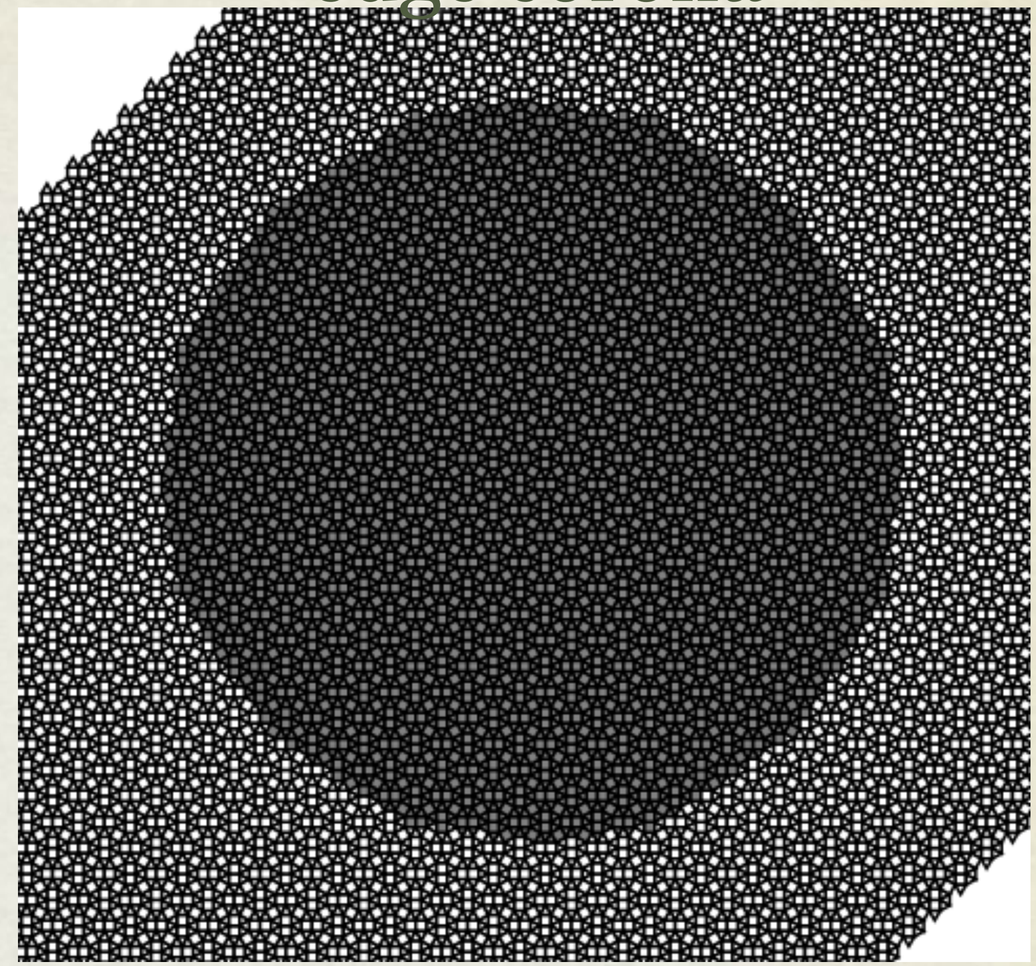

1.5
1.0
0.5
- 2
- 1
1
2
- 0.5
- 1.0
- 1.5



corona

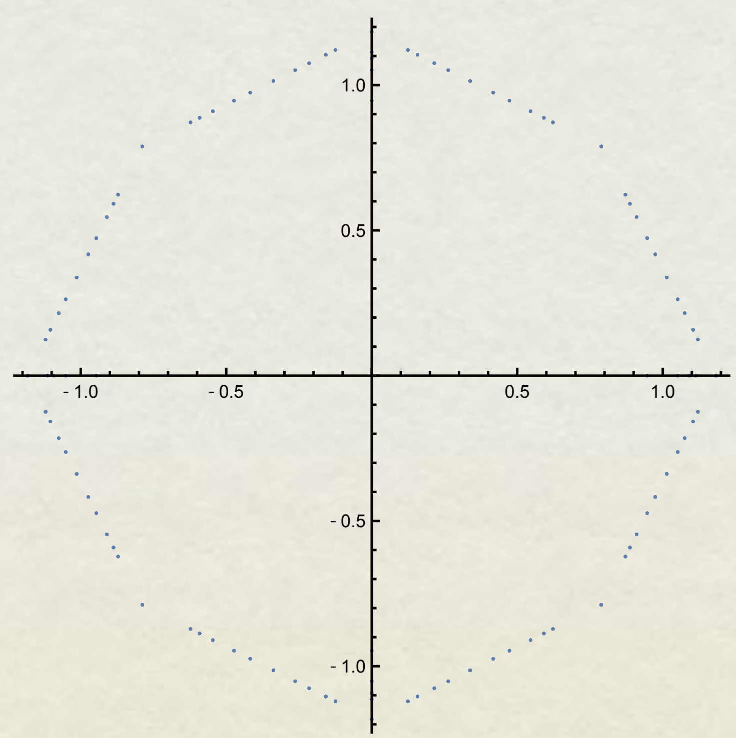


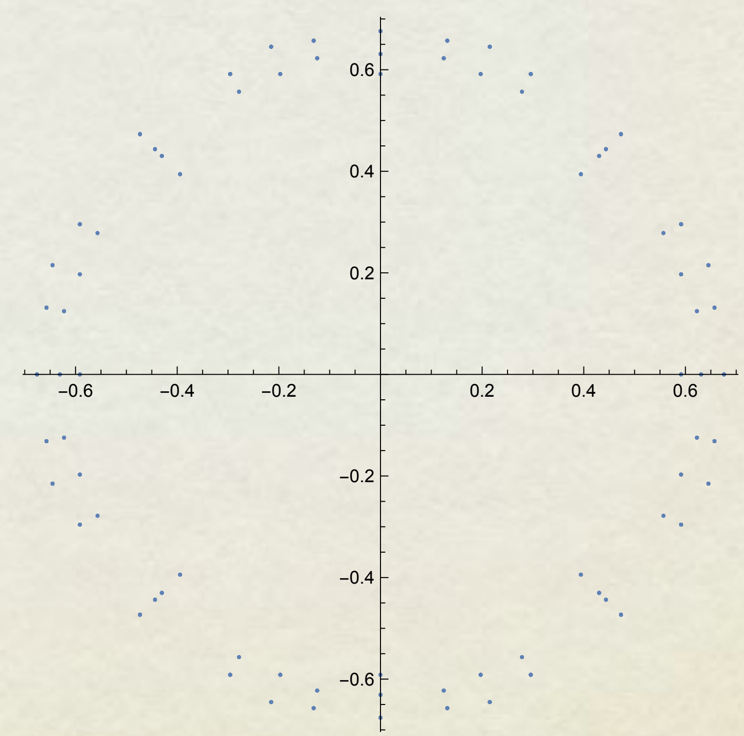
edge corona

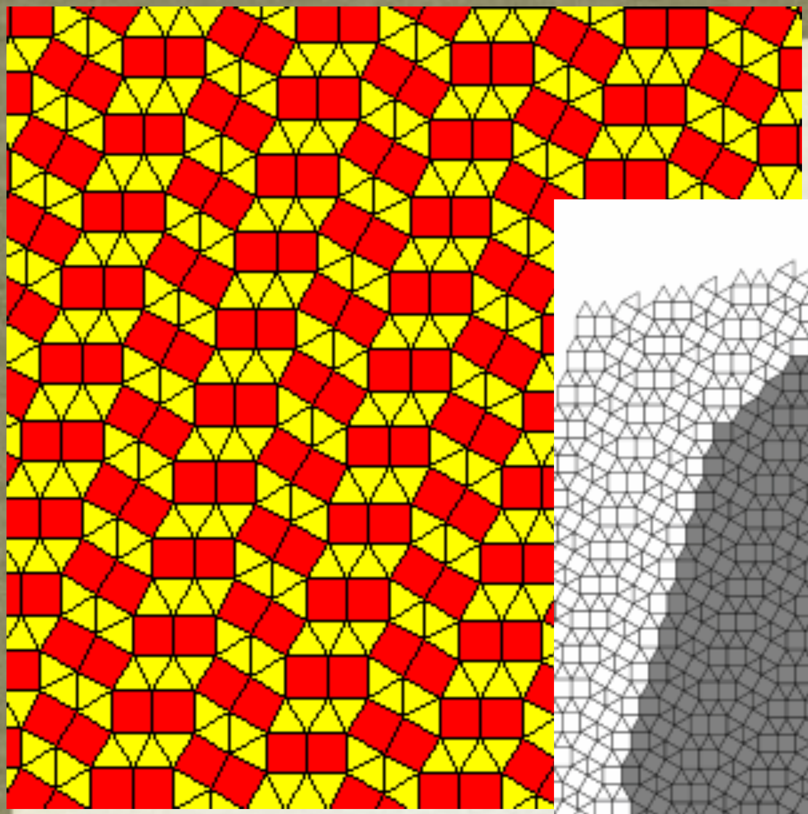


octagon

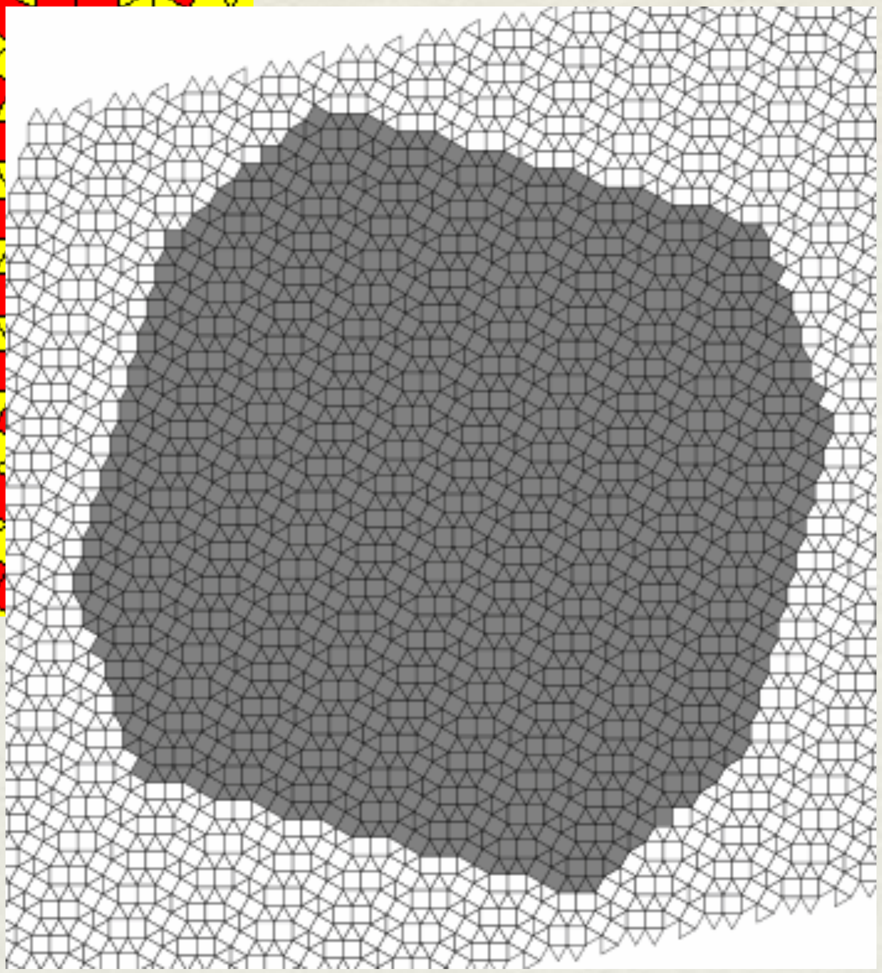
hexadecagon



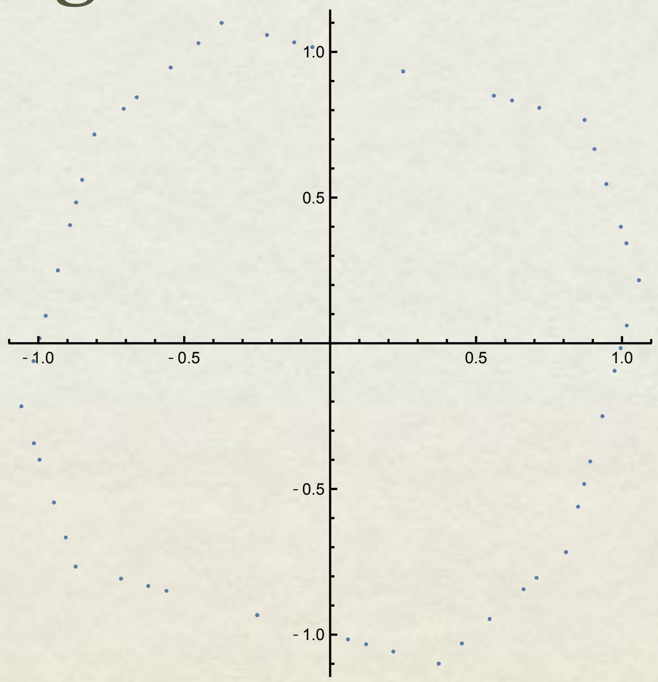





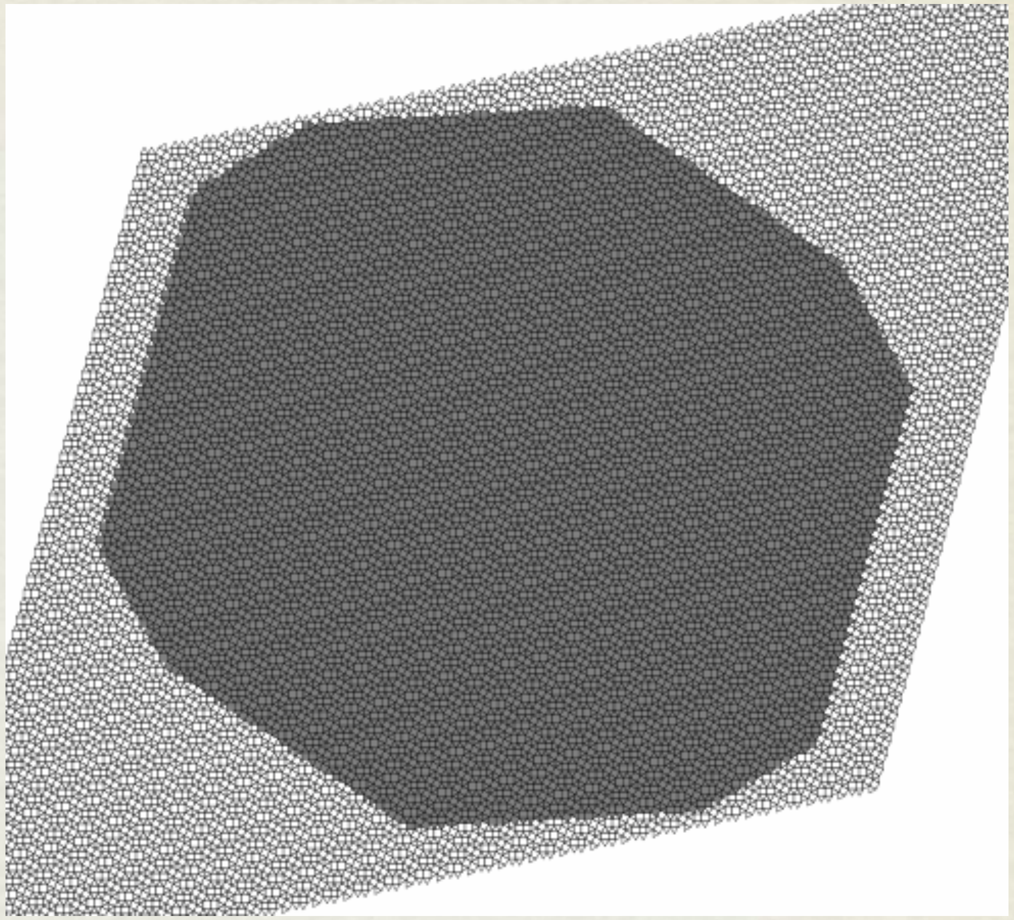
## corona



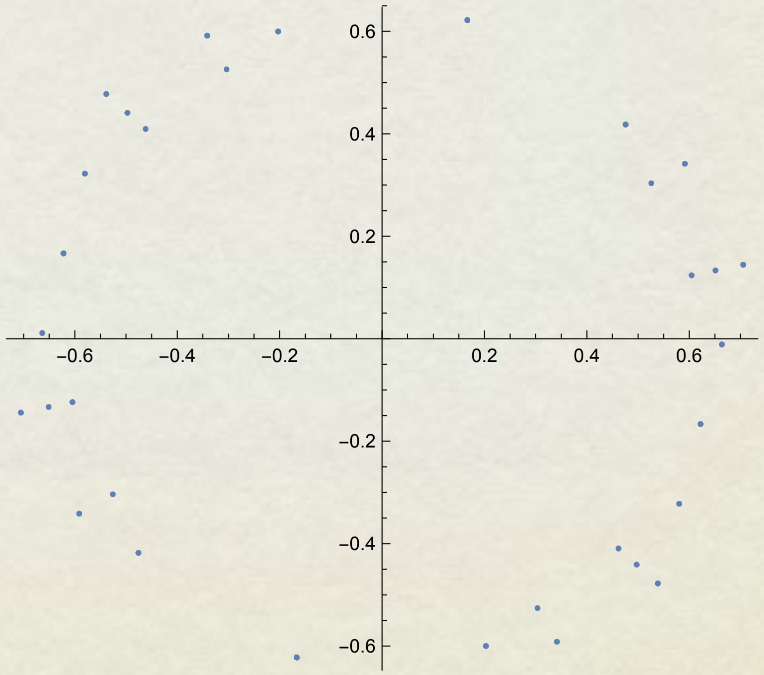
## octagon




## edge corona

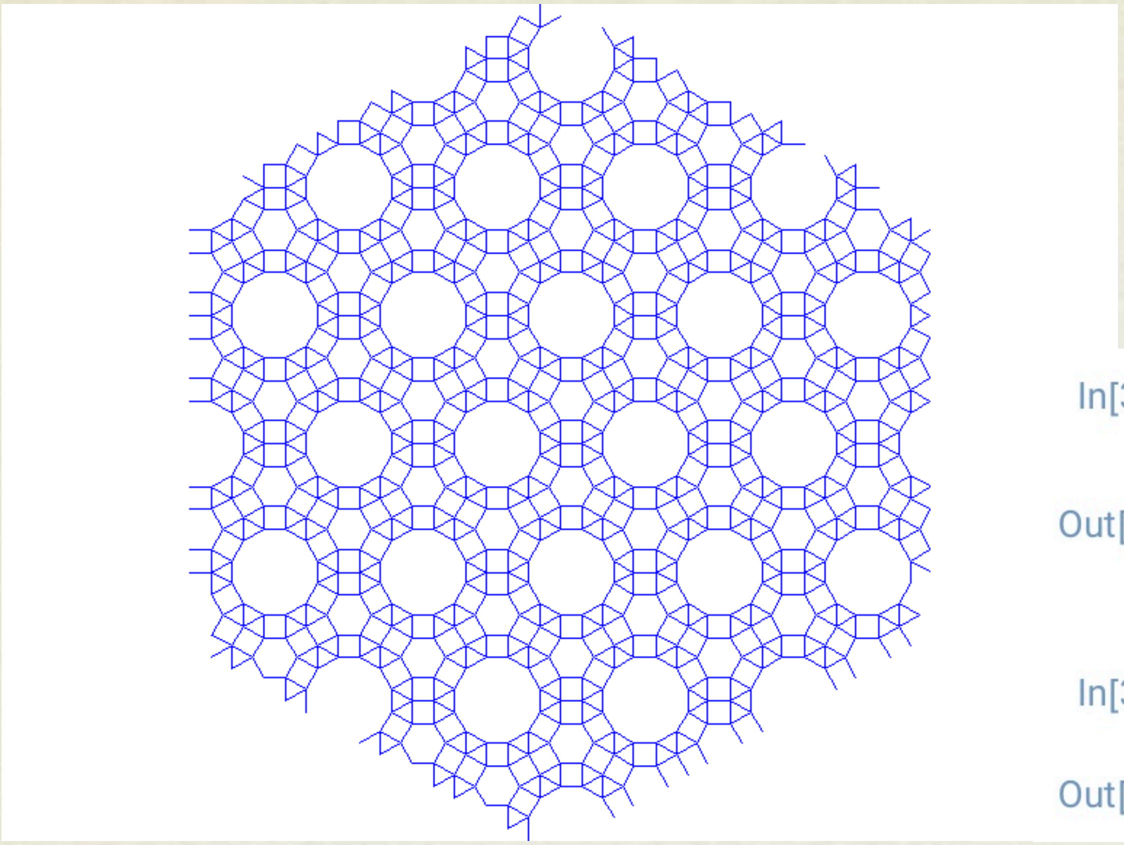


## decagon



# 3-uniform #35

Brian Galebach http://probabilitysports.com/tilings.html

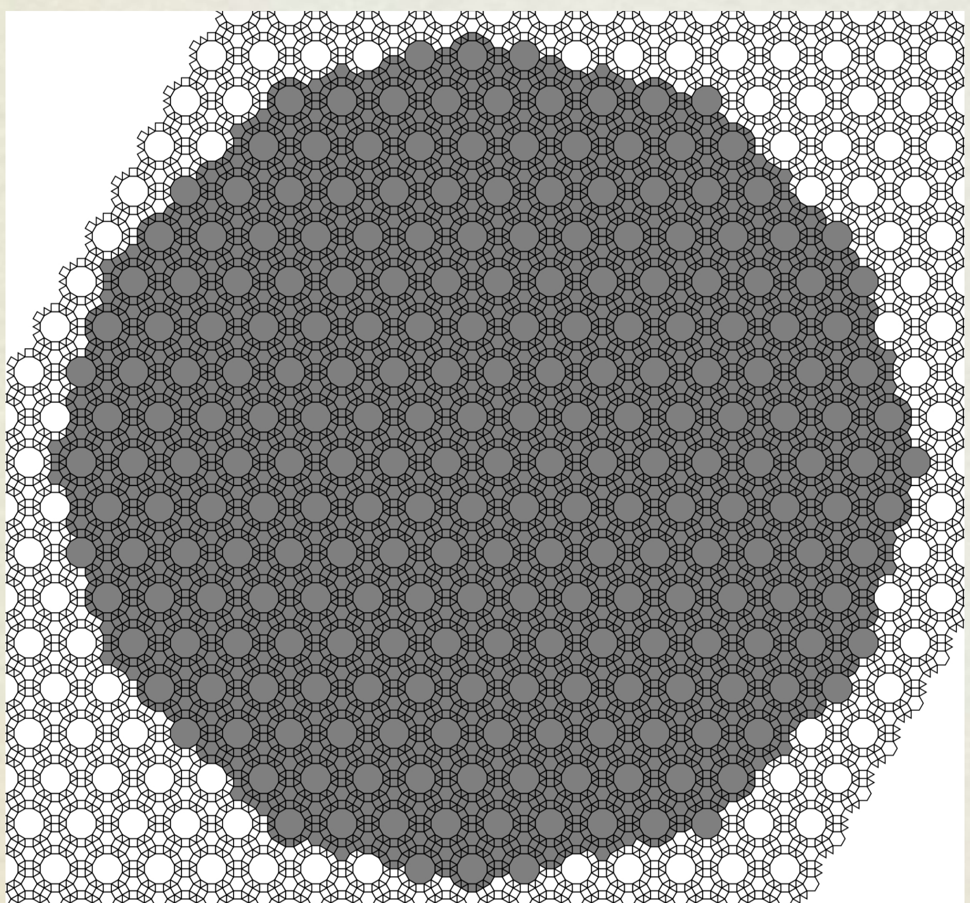


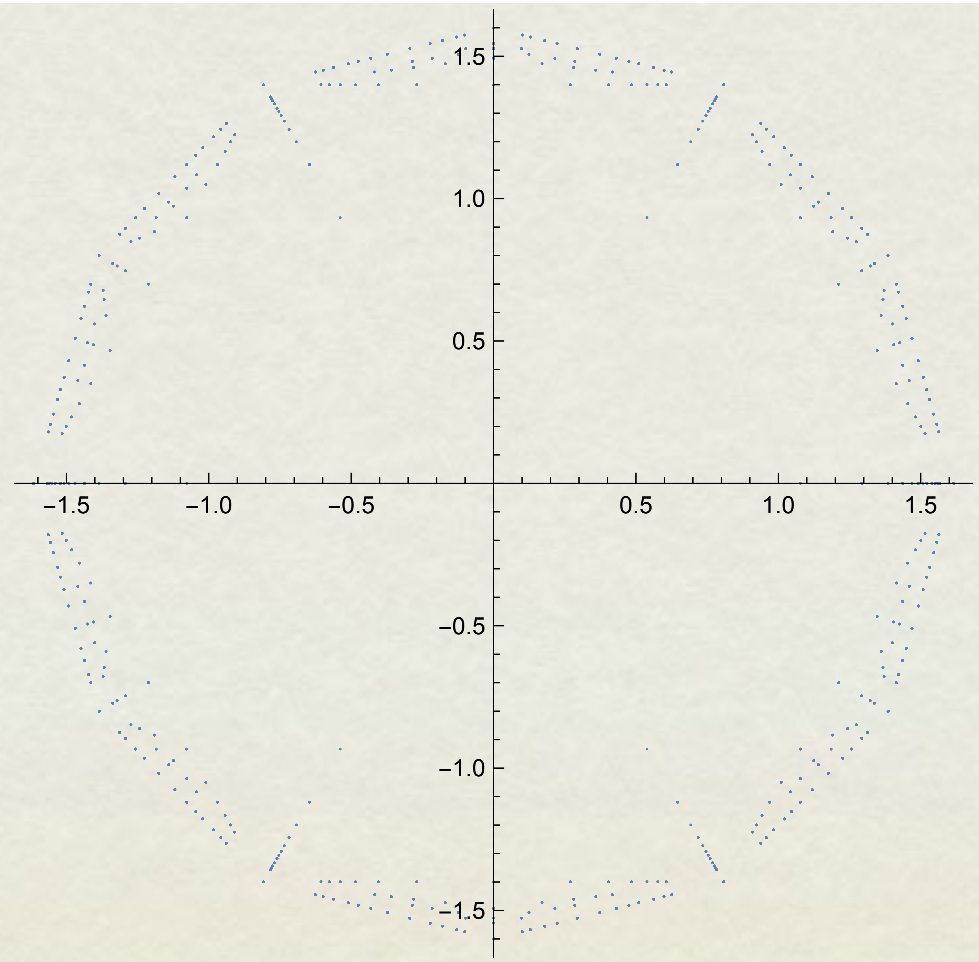
In[38]:= **FullSimplify[Map[Norm, v]]**

Out[38]= $\left\{\frac{1}{4}\left(3+2\sqrt{3}\right), \frac{1}{4}\left(3+2\sqrt{3}\right), \frac{1}{4}\left(3+2\sqrt{3}\right), \frac{3}{7}\left(2+\sqrt{3}\right),\right.$

In[39]:= **N[%]**

Out[39]= {1.61603, 1.61603, 1.61603, 1.59945, 1.59945,

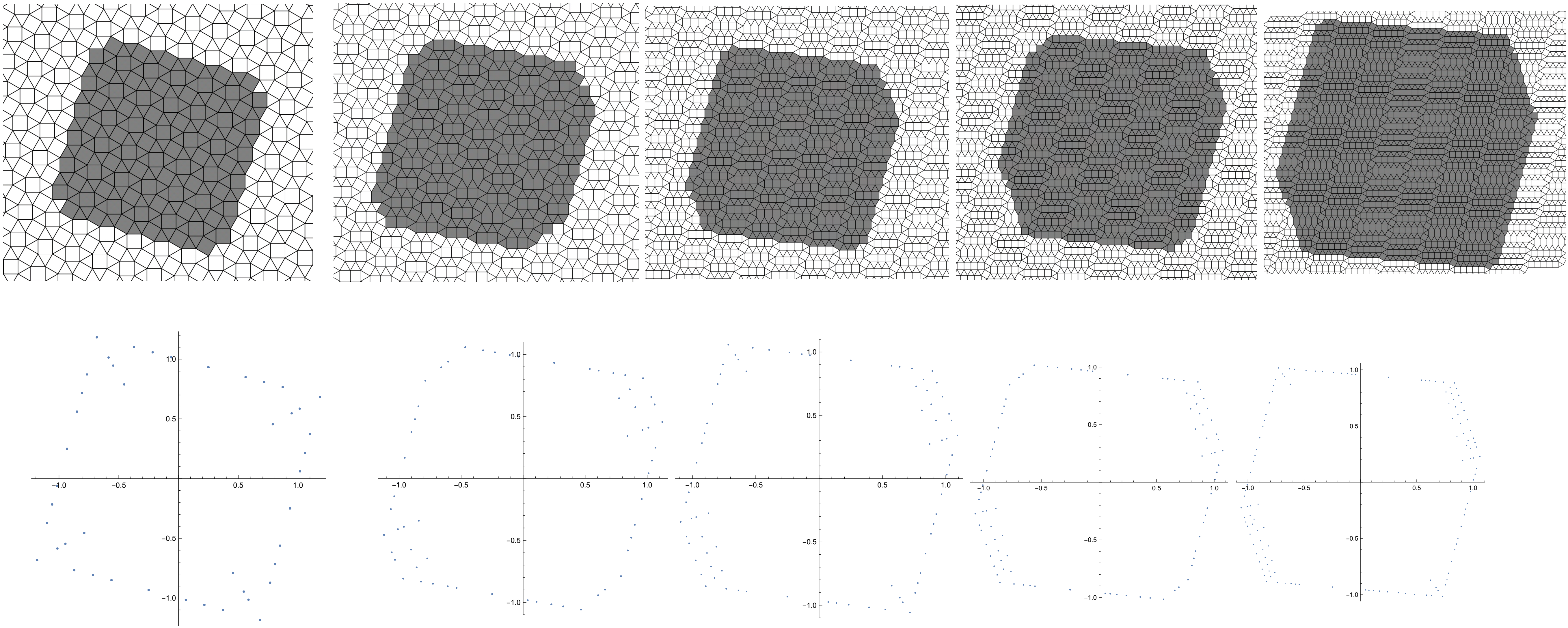




# THE NUMBERS OF EDGES

| | # | Corona | Edge corona |
|---|---|---|---|
| 1-uniform | 11 | 4, 6 | 4, 6, 8 |
| 2-uniform | 20 | 4, 6, 8, 12 | 4, 6, 10, 16 |
| 3-uniform | 61 | 4, 6, 8, 10, 12 | |

How's the upper bound of the number of edges in the case of regular tiling?

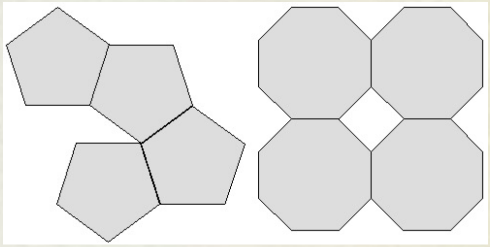


1.0
0.5
-1.0
-0.5
0.5
1.0
-0.5
-1.0

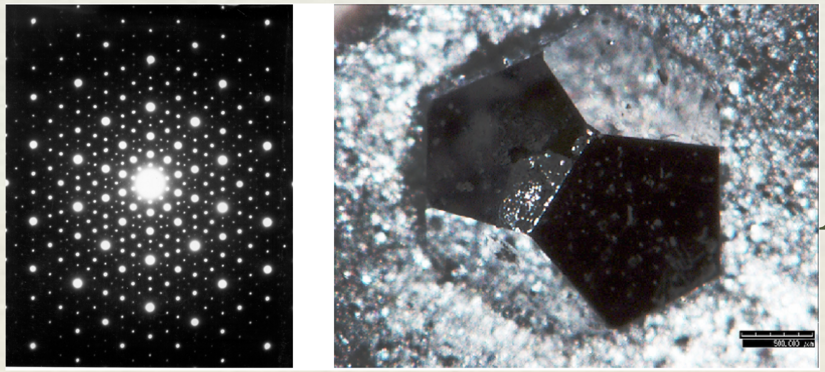
# CRYSTALLOGRAPHIC RESTRICTION THEOREM

https://en.wikipedia.org/wiki/Crystallographic_restriction_theorem

The crystallographic restriction theorem in its basic form was based on the observation that <u>the rotational symmetries of a crystal are usually limited to 2-fold, 3-fold, 4-fold, and 6-fold</u>.



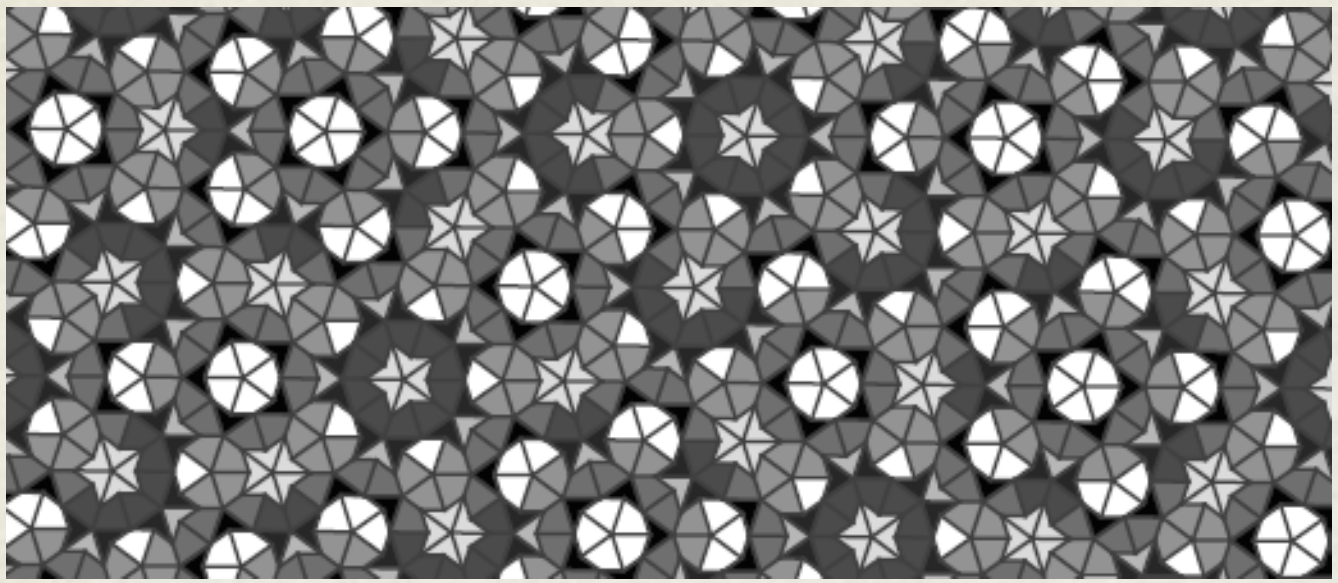
However, <u>quasicrystals</u> can occur with other diffraction pattern symmetries, such as <u>5-fold</u>; these were not discovered until 1982 by Dan Shechtman.



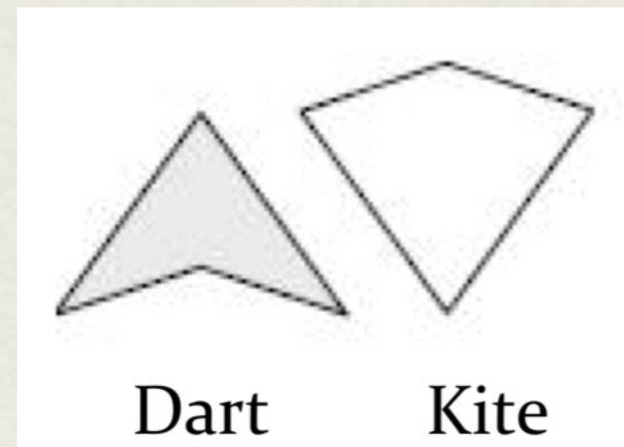
A.P. Tsai

# PENROSE TILINGS

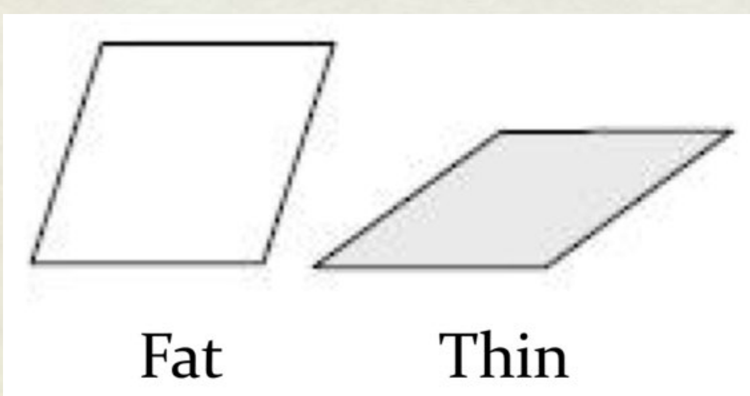
(Penrose 1970's)

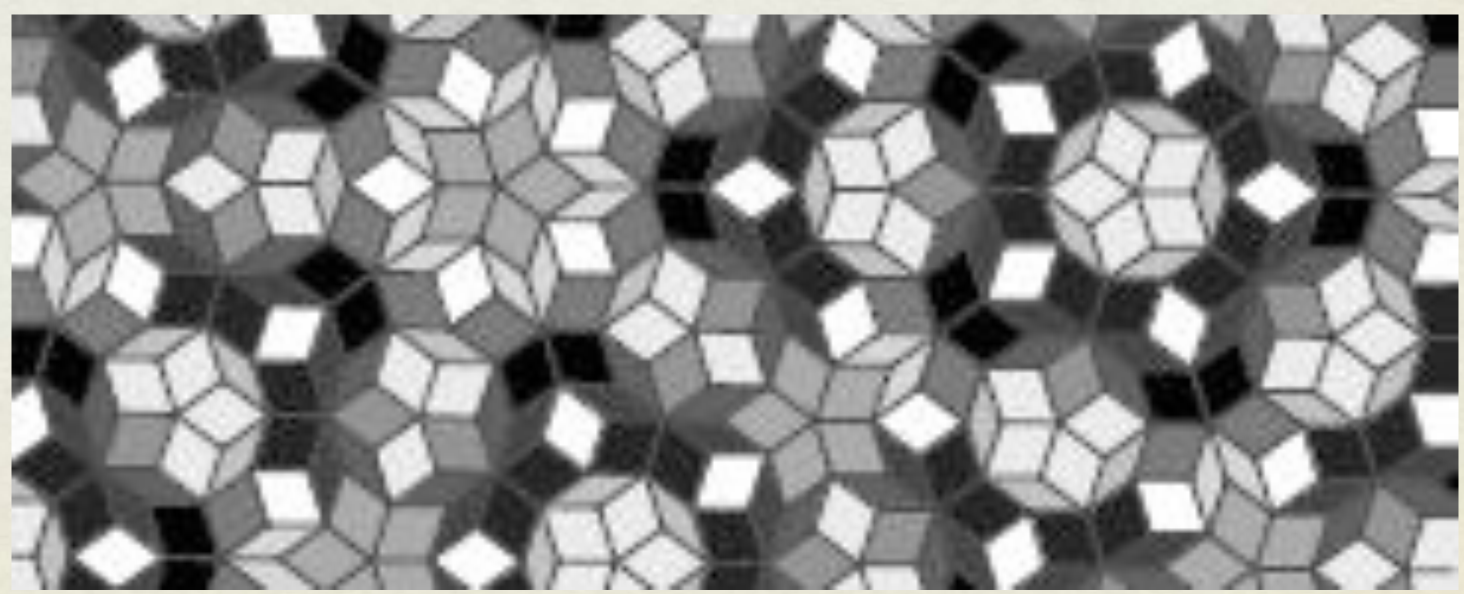


Kite and Dart (P2)

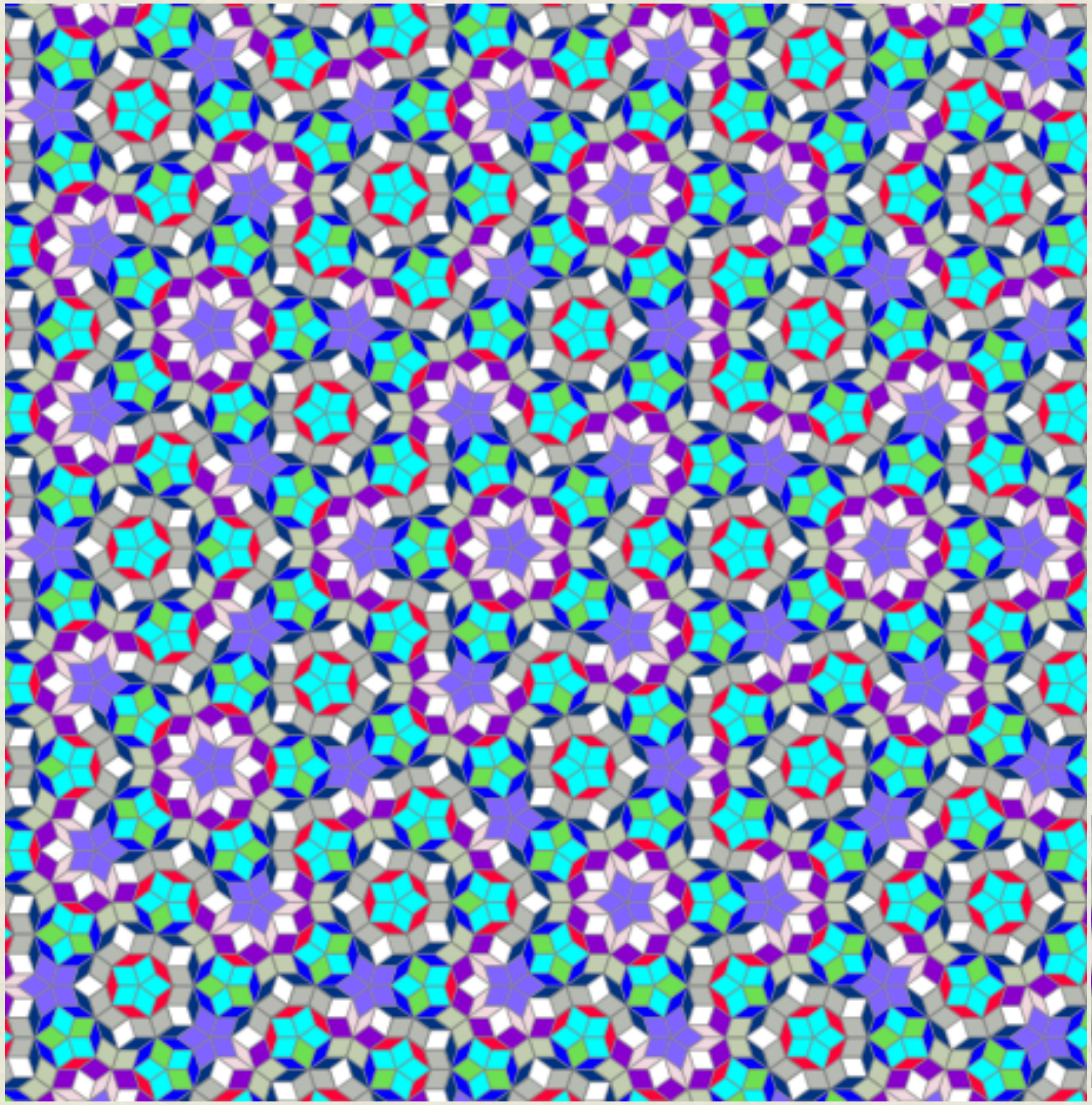


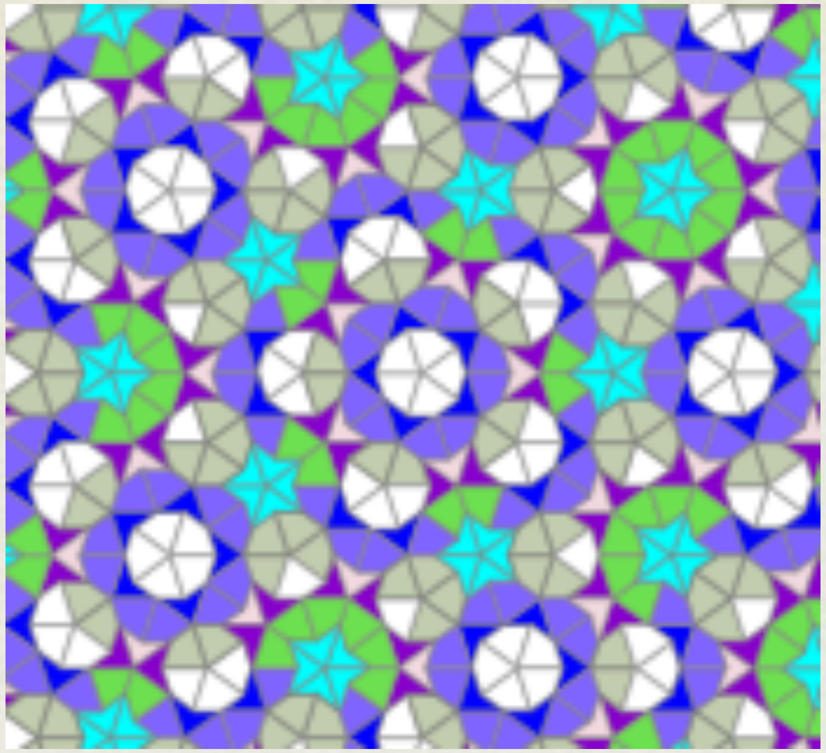

Rhombus (P3)

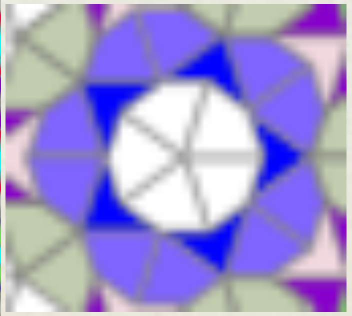


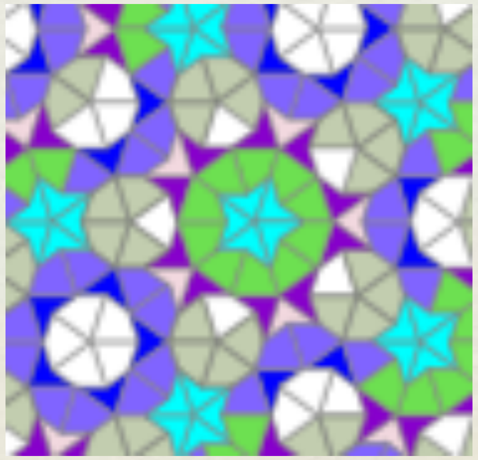



# FIVE FOLD LOCAL SYMMETRY



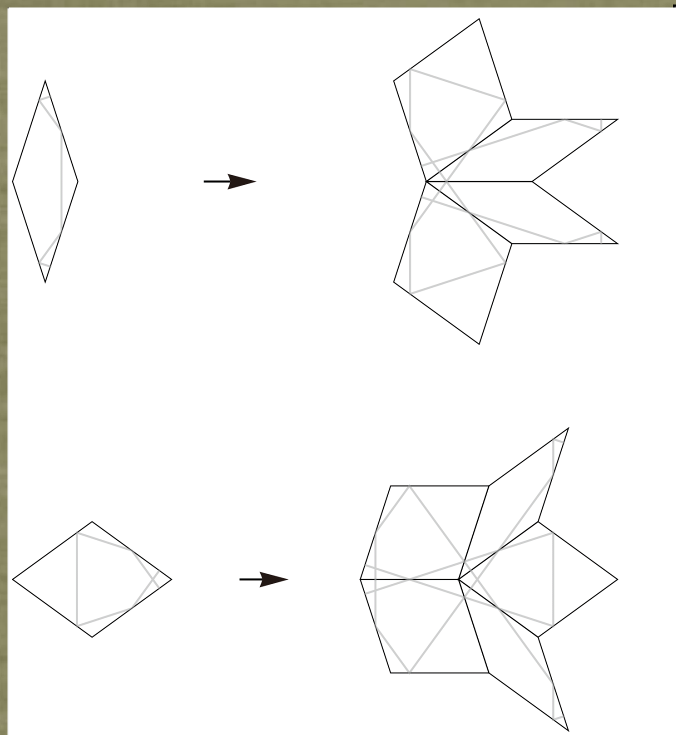


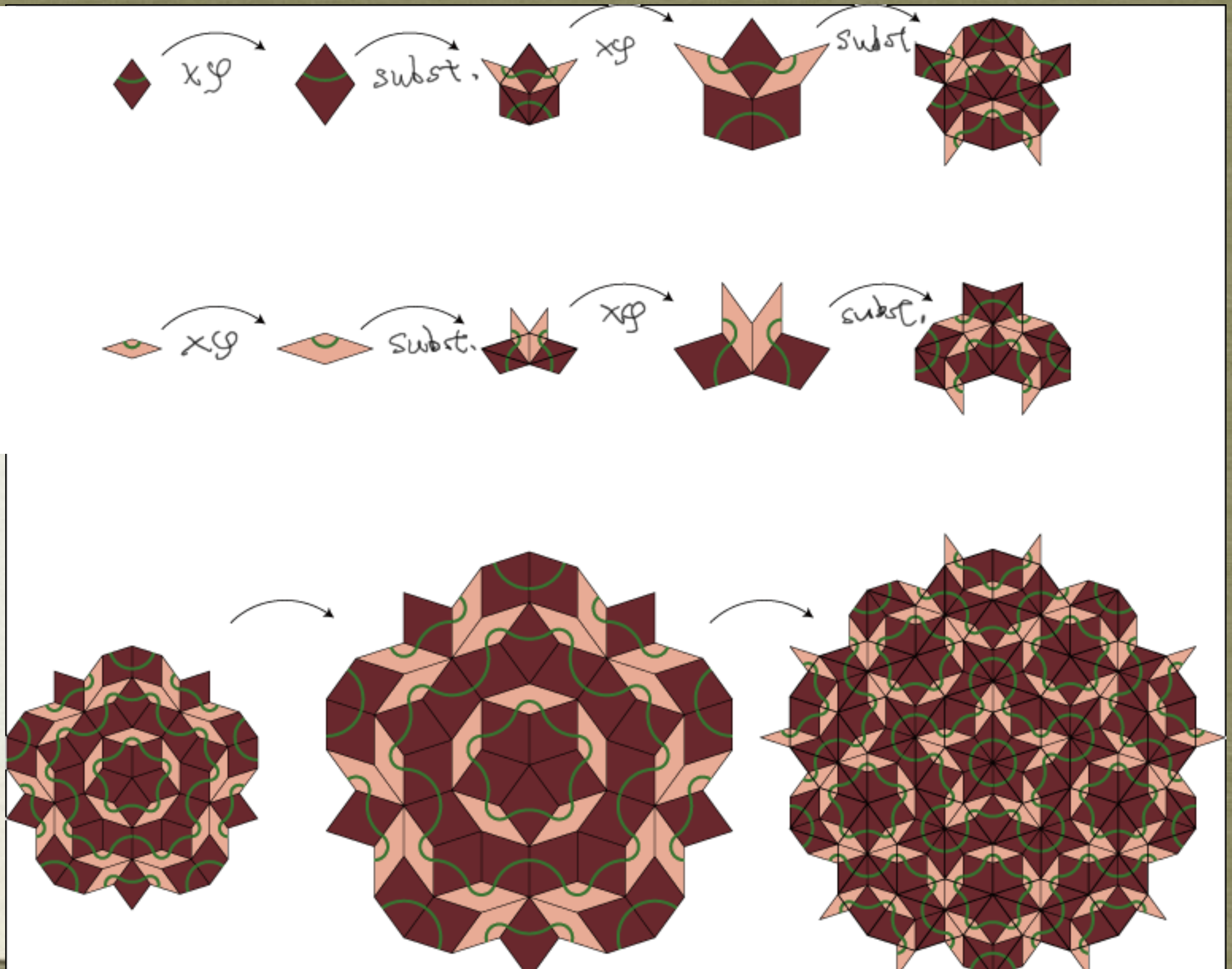




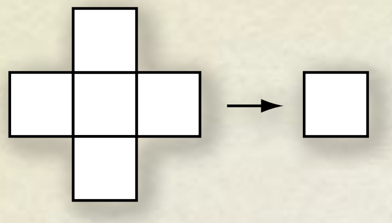
72-degree rotation symmetry does not agree with translational symmetry.

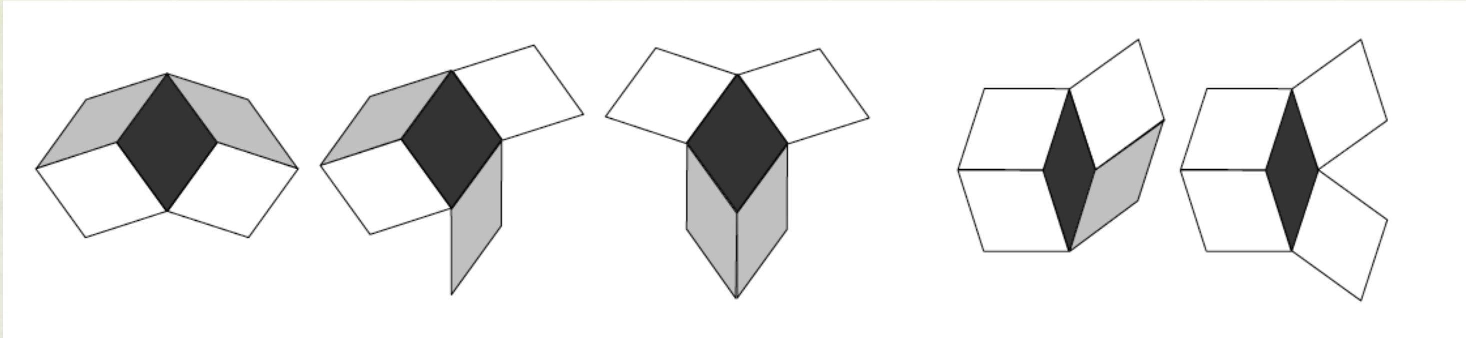
# TILING BY SUBSTITUTIONS



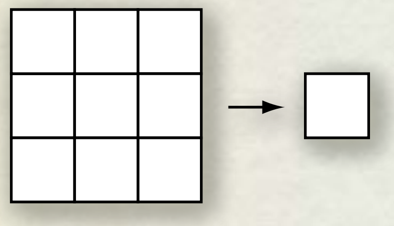


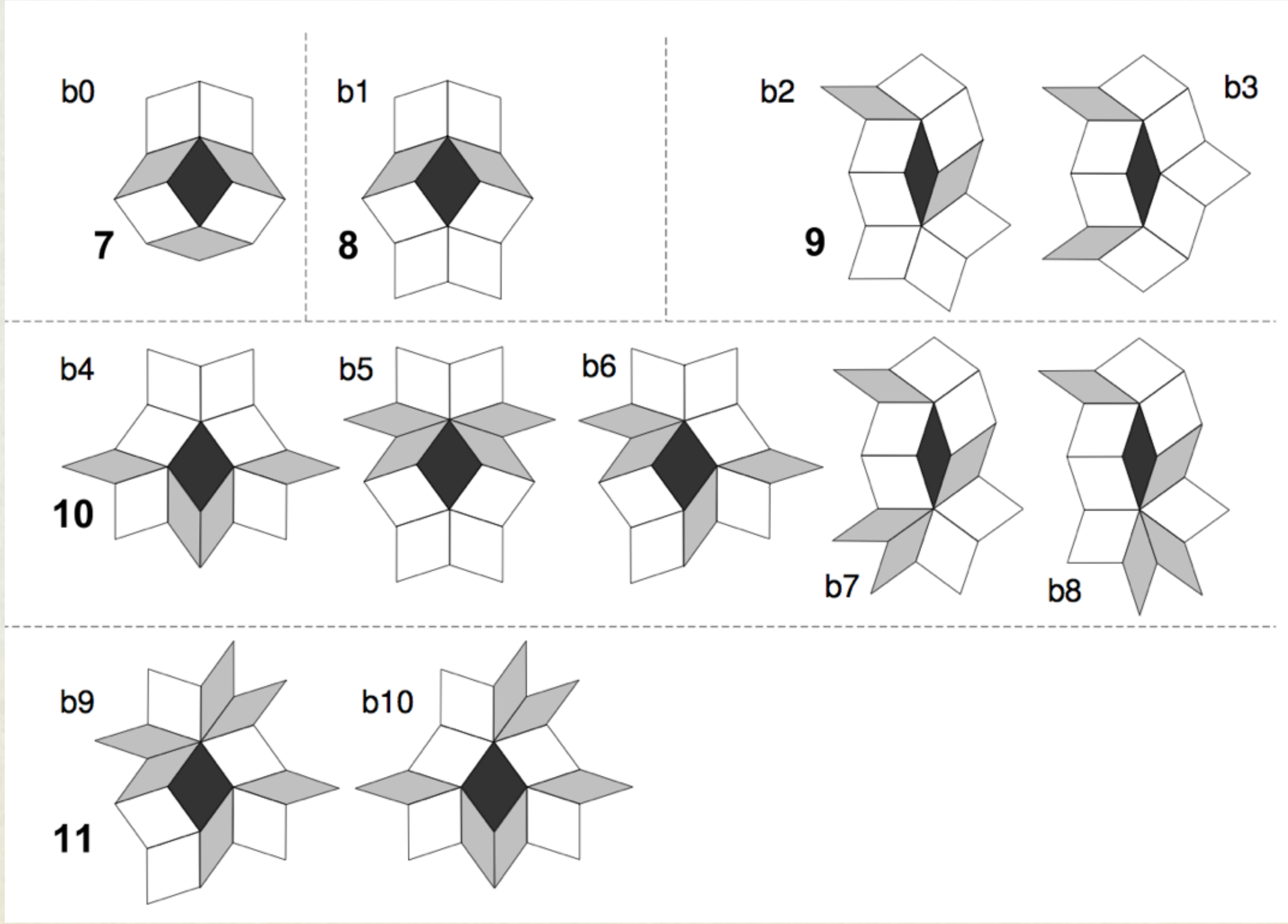
# NEIGHBORHOODS





(Owens, Stepney 2008)

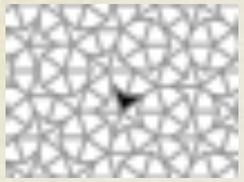




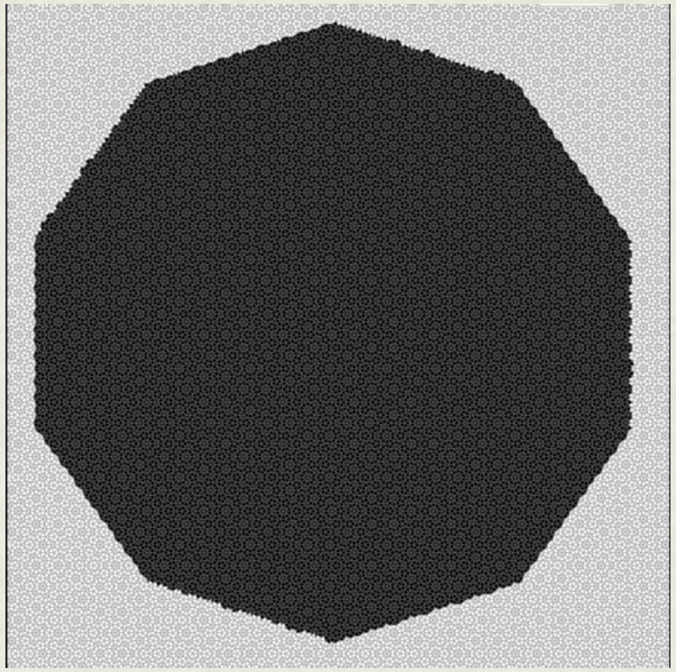
# RADIUS 60

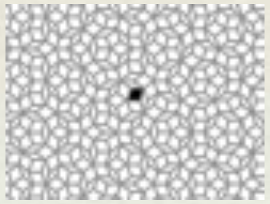
Kite Dart, von Neumann

Rhomb, Moore

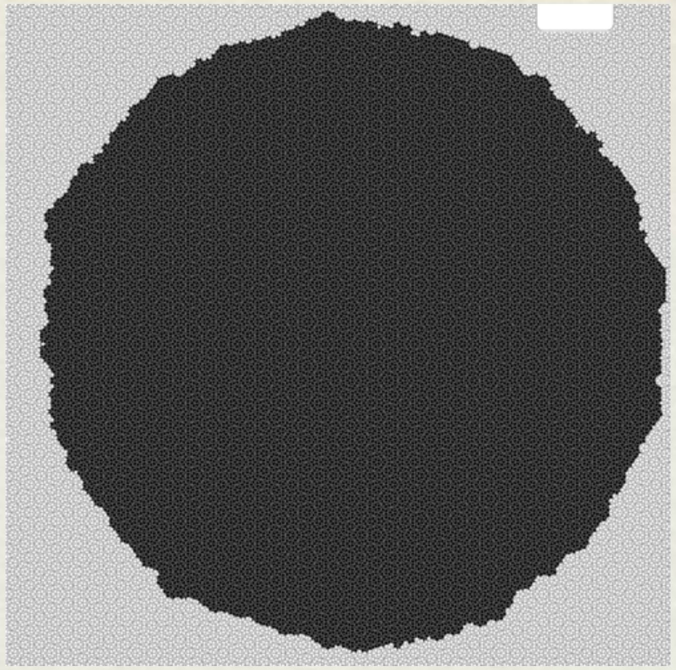


a dart





a fat



Scale the figure by the number of steps → limit shape exists?

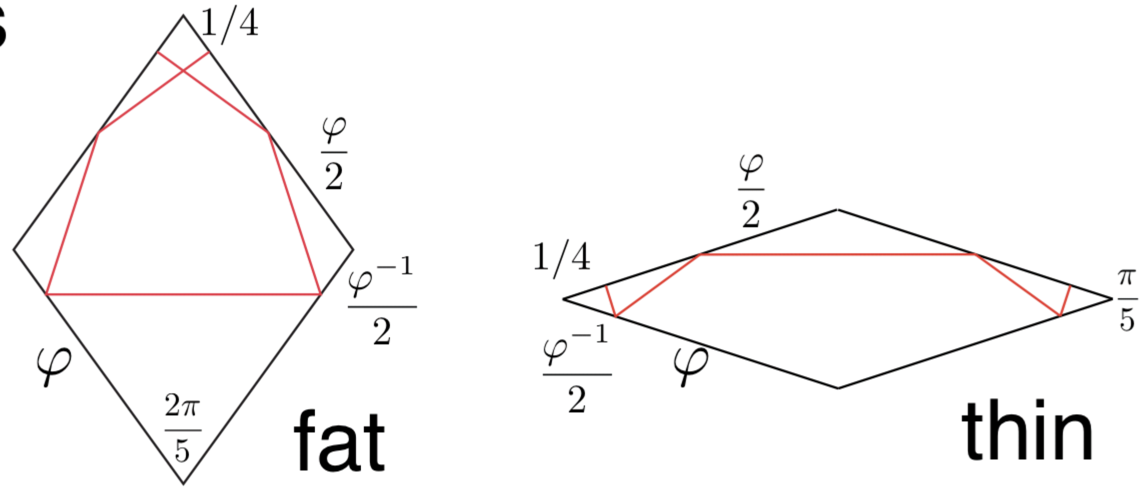
**Any initial patch ⇒ regular decagon ??**

The corona limit of Penrose tilings is a regular decagon.

Akiyama, Imai 2016

# AMMANN BARS

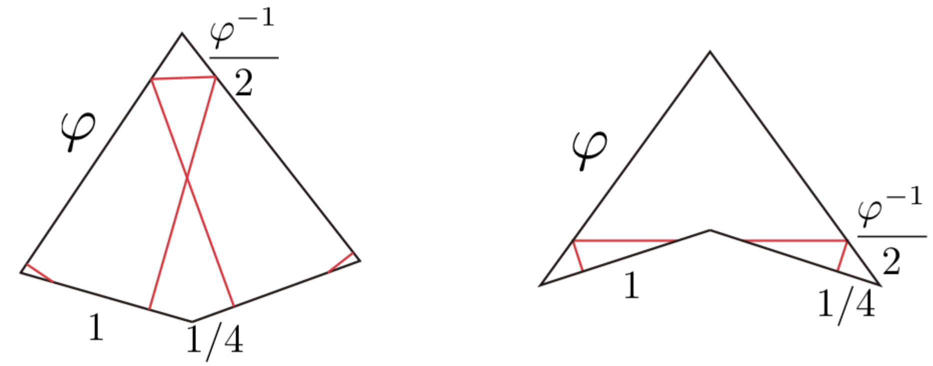
rhombus

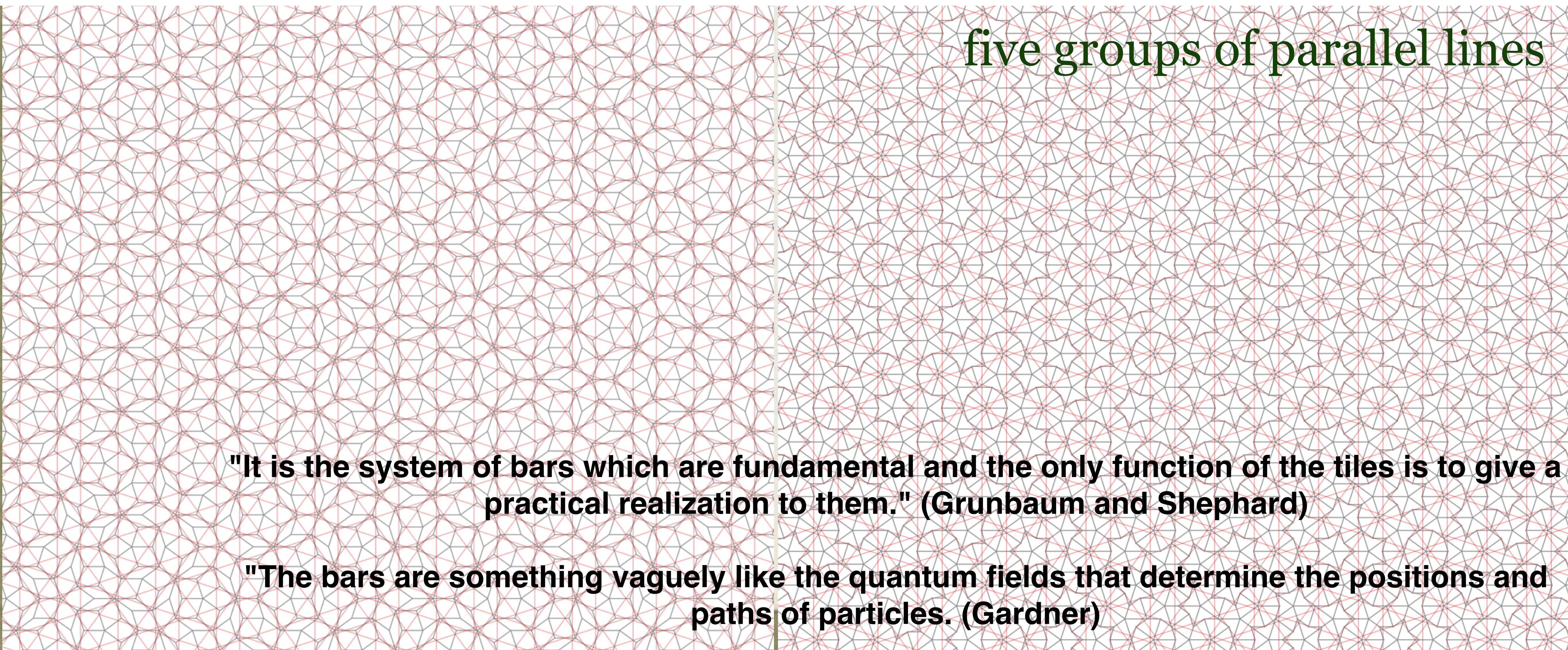



fat

thin

kite and dart



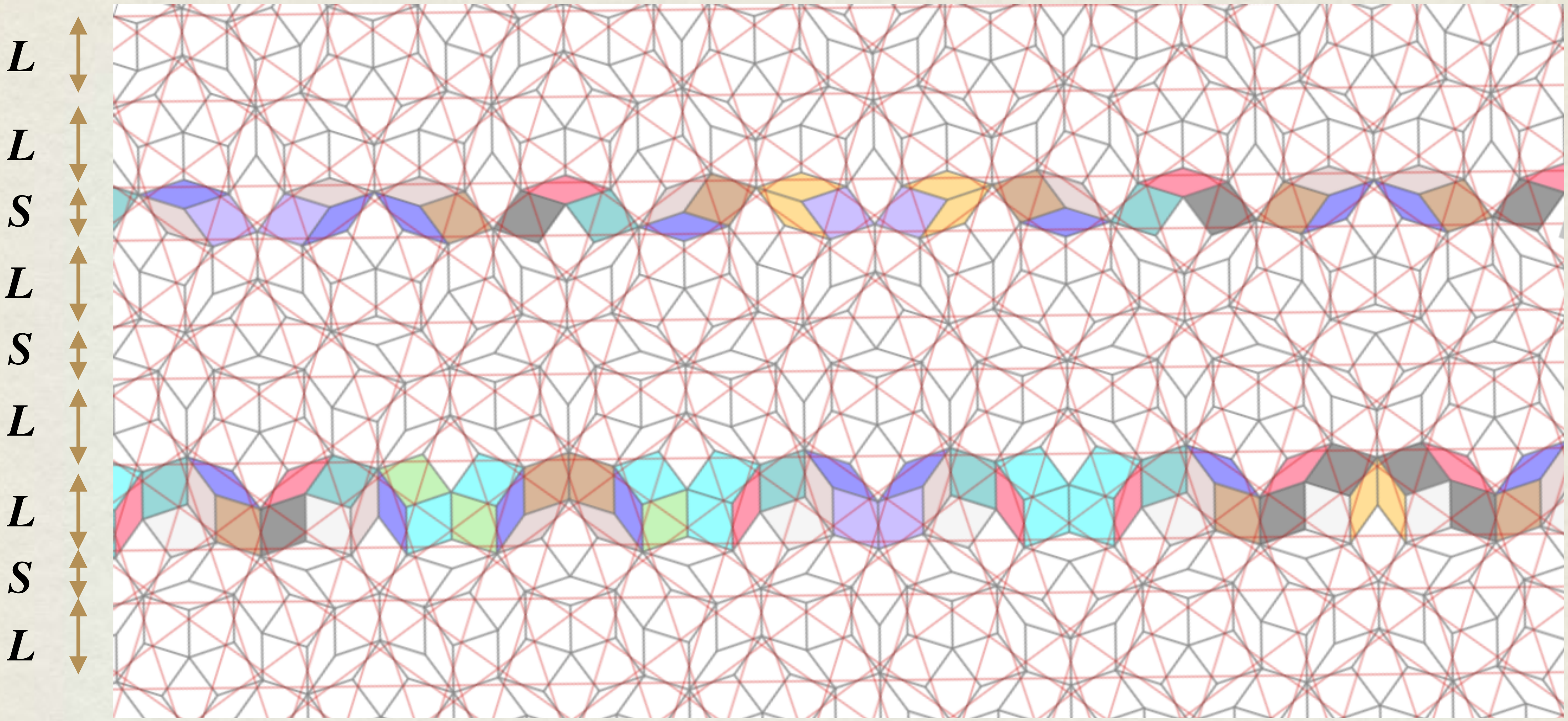



five groups of parallel lines

**"It is the system of bars which are fundamental and the only function of the tiles is to give a practical realization to them." (Grunbaum and Shephard)**

**"The bars are something vaguely like the quantum fields that determine the positions and paths of particles. (Gardner)**

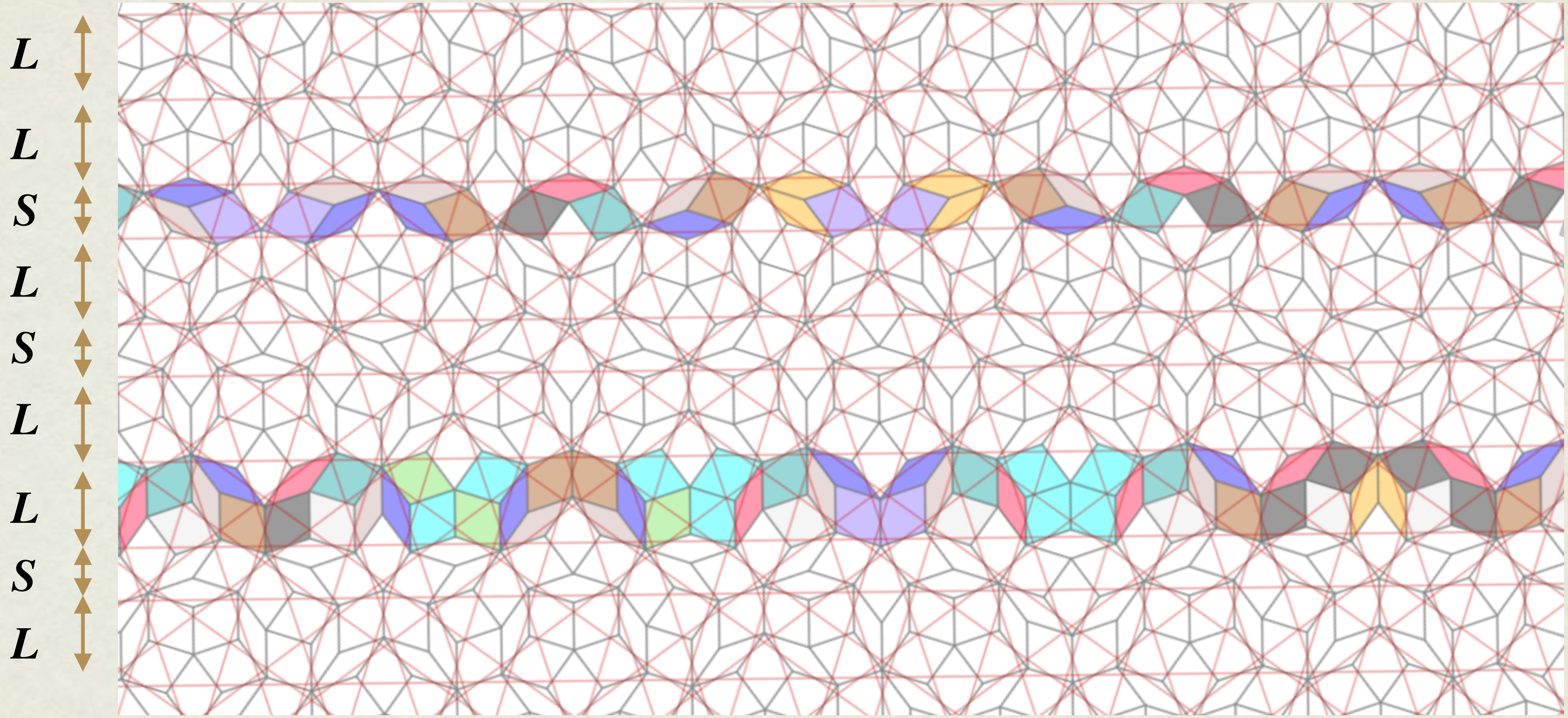
## Propagating speed: $l/2$ per step, $l \in \{L, S\}$




| | rhombus | kite and dart |
|---|---|---|
| von Neumann | $L/2,\ S/2$ | $L/3,\ S/2$ |
| Moore | $L,\ S$ | $L/2,\ S$ |

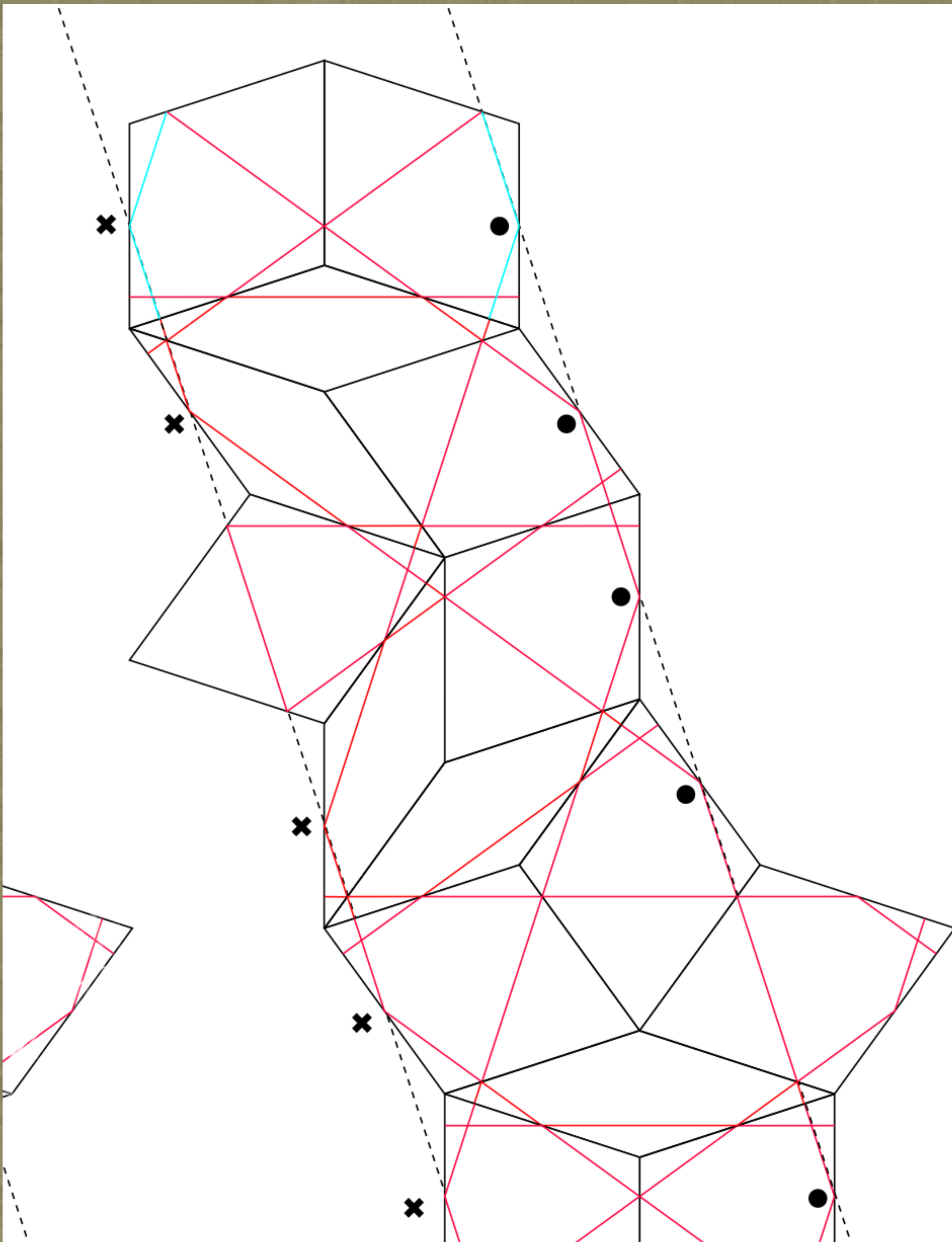
# PROPAGATING SPEED BETWEEN TWO AMMAN BARS

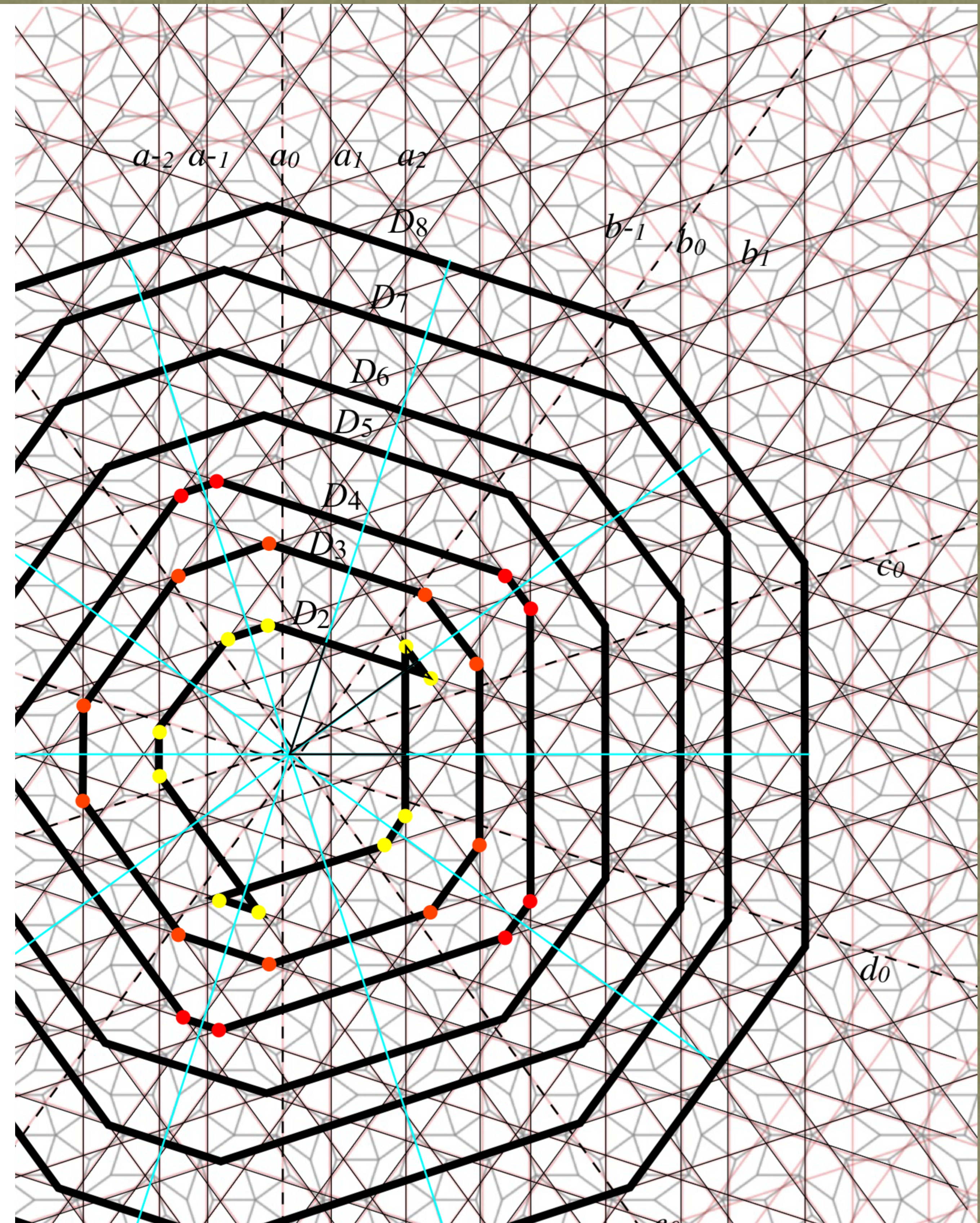
Propagating speed: $l/2$ per step, $l \in \{L, S\}$




The ratio of occurrence of $L$ and $S$ → the golden ratio.

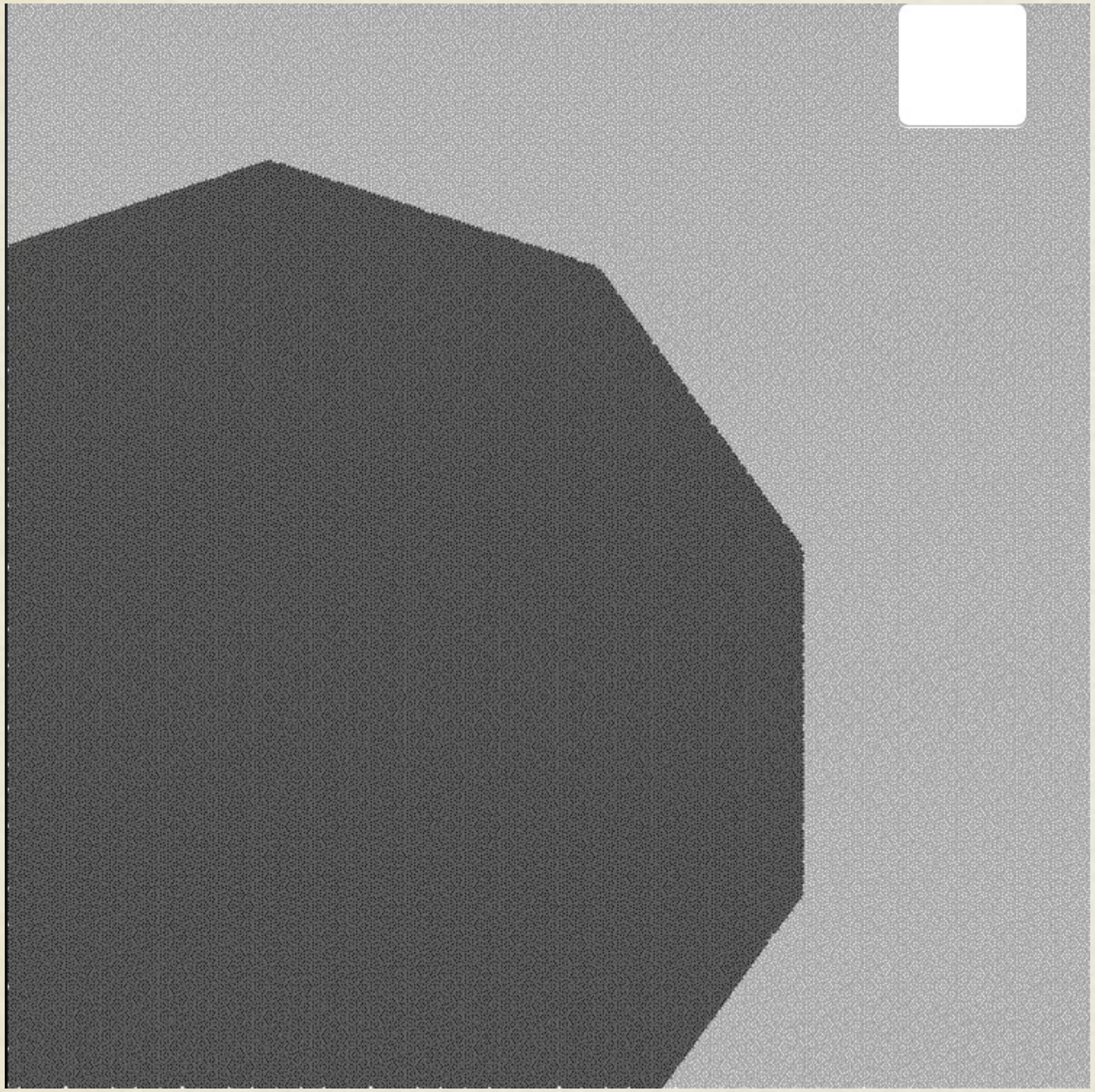
# DECAGONS & GAPS



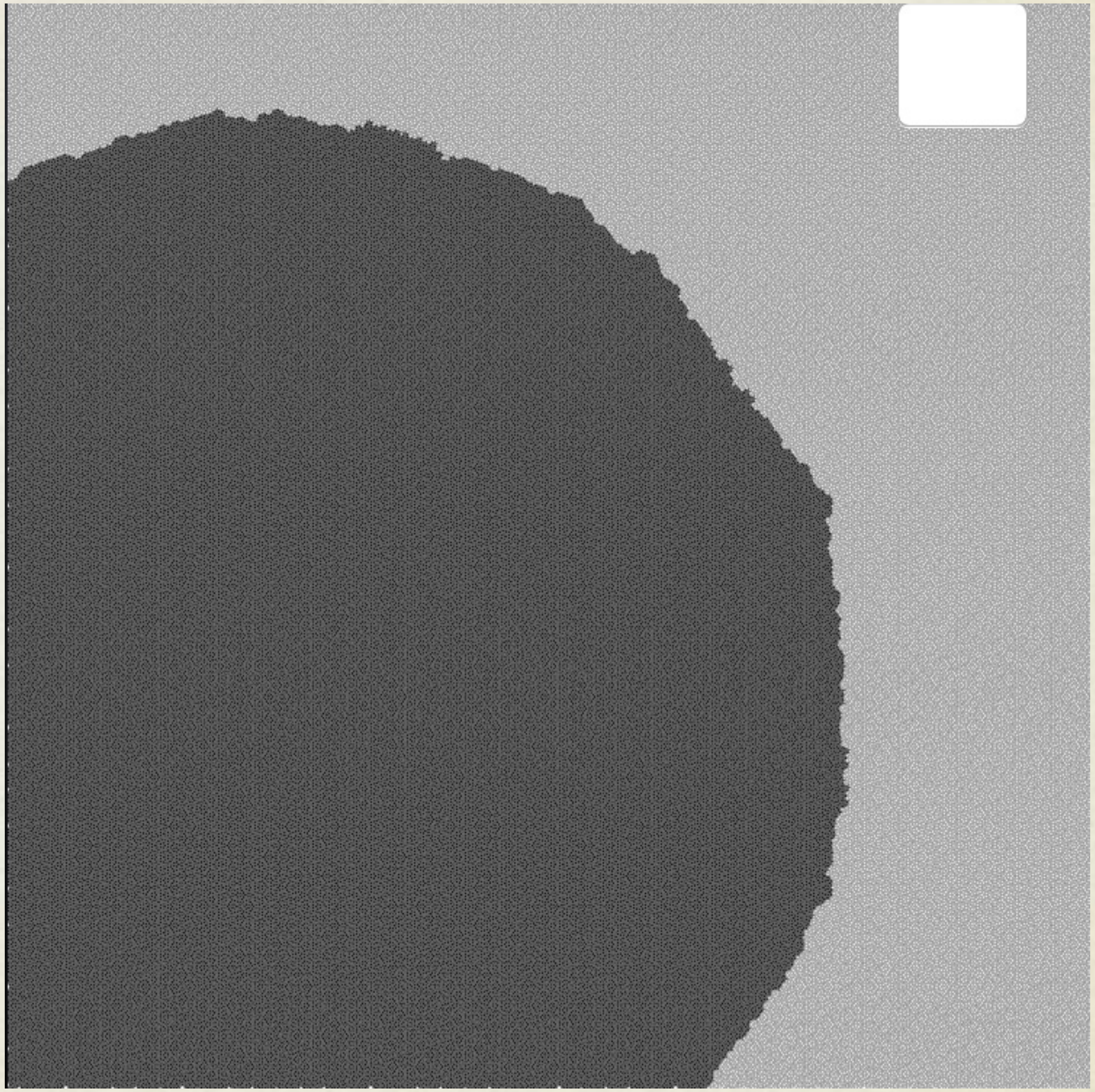


# EFFECT OF THE DIFFERENCE OF NEIGHBORHOOD

Rhomb, von Neumann



Rhomb, Moore



**Why the speed of convergence differ?**

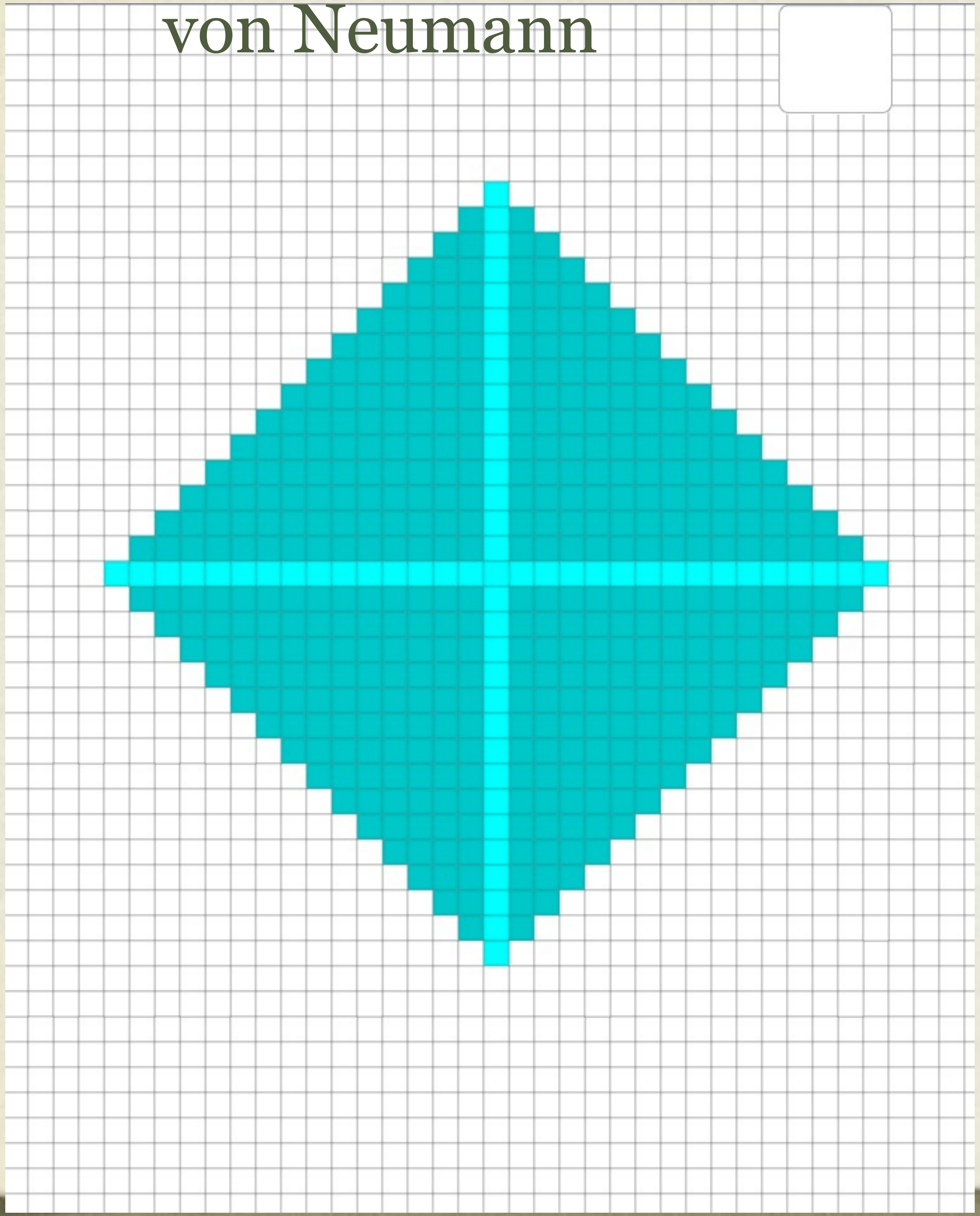
# MODIFIED GROWTH RULE

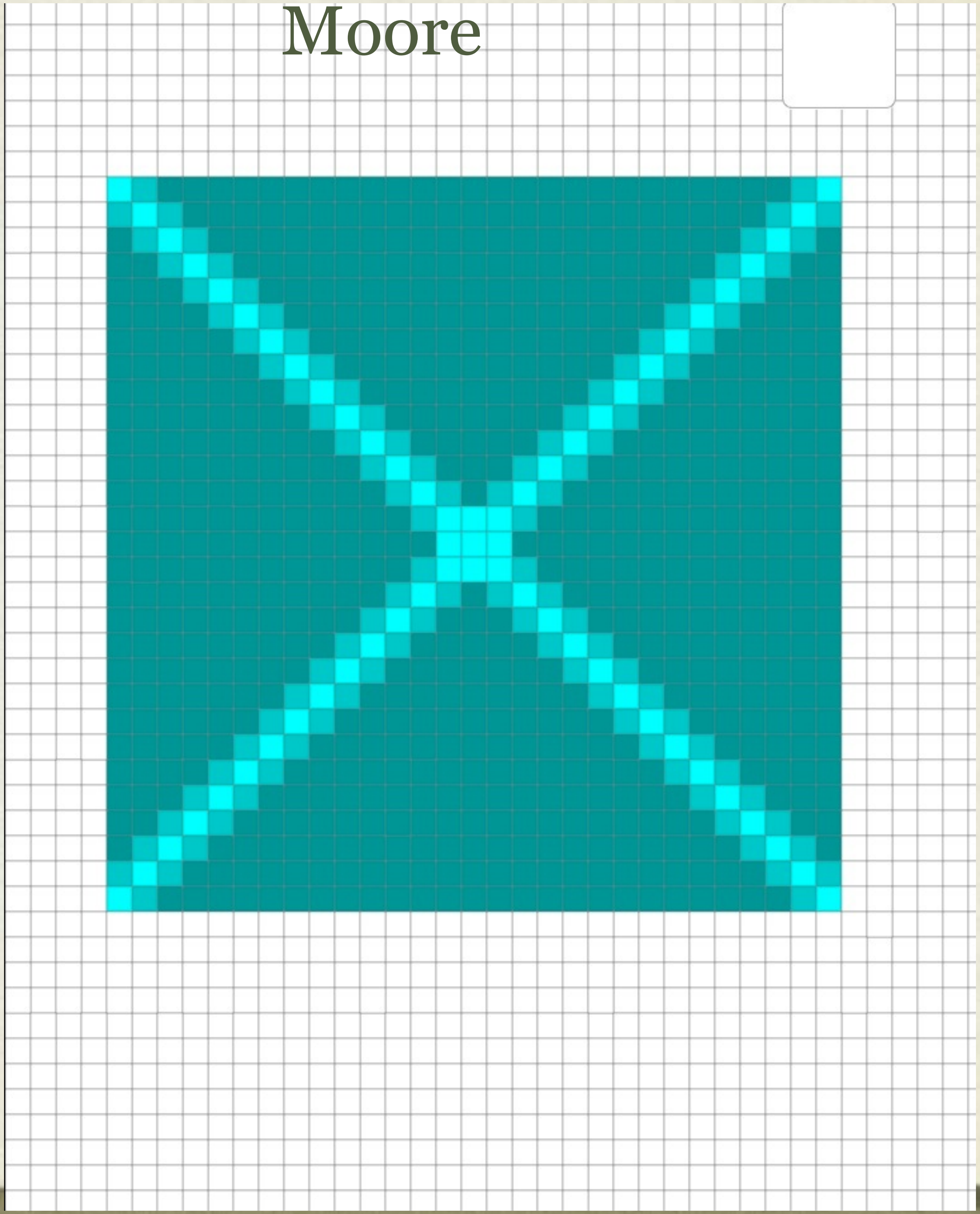
State set= {0,1,2,3}
Rule: focus cell = 0:
    case the sum of non-zero neighborhood cells is
        1 then the next state is 1;
        2 then the next state is 2;
        3 or more then the next state is 3;
otherwise: keep its state)

**Counting arrival signals...**

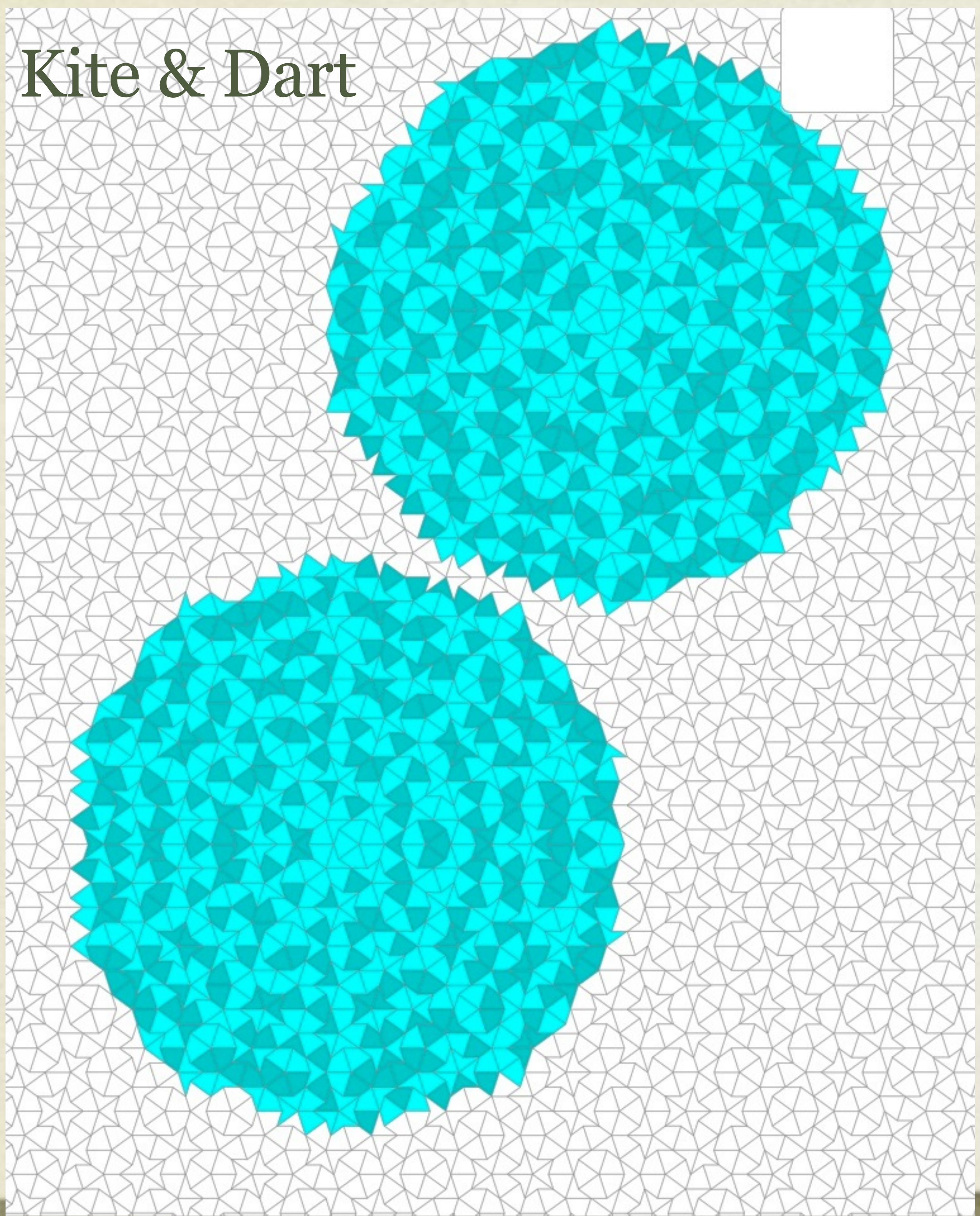
Init conf.: a "1" cell

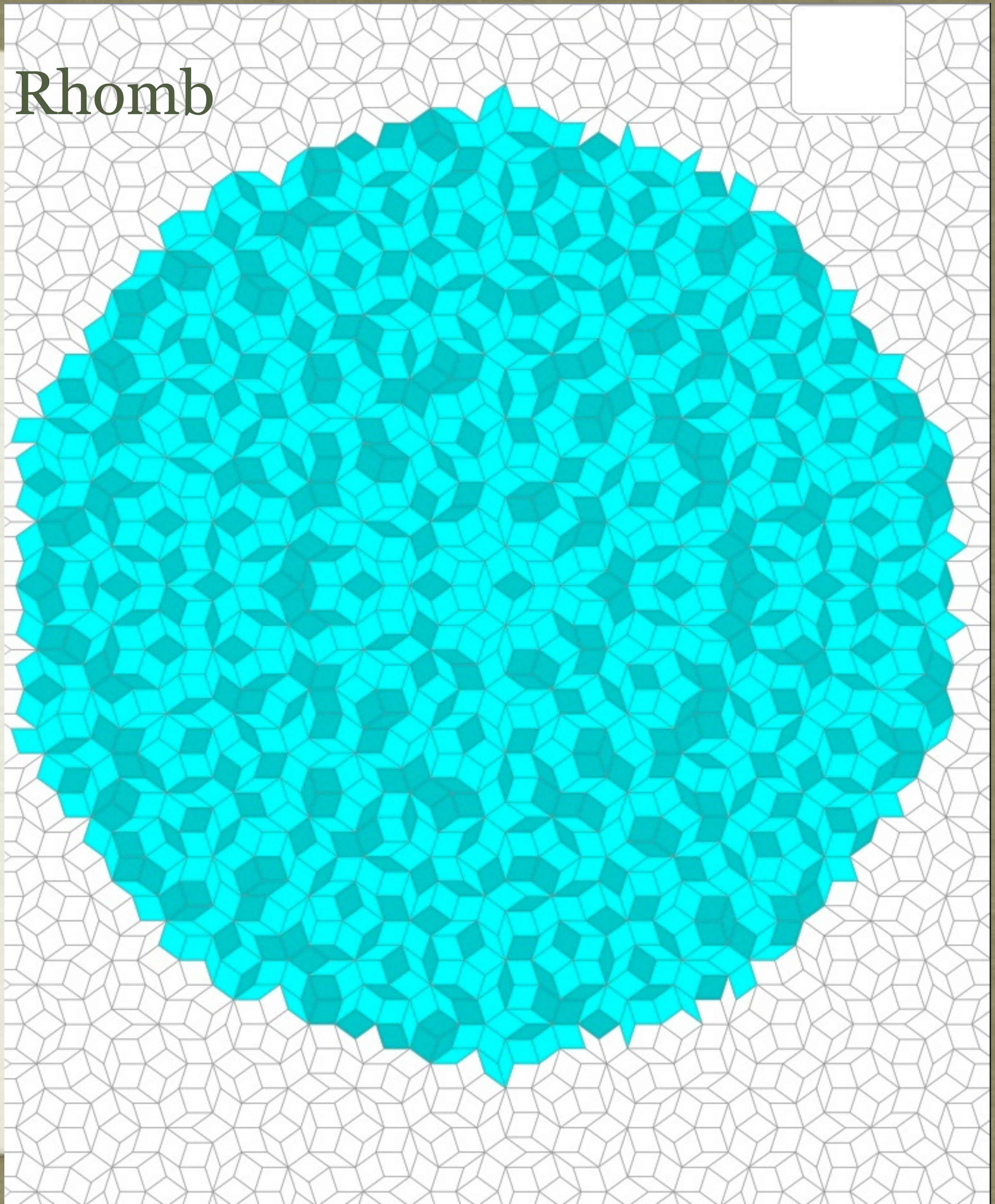
# SQUARE LATTICE CASE



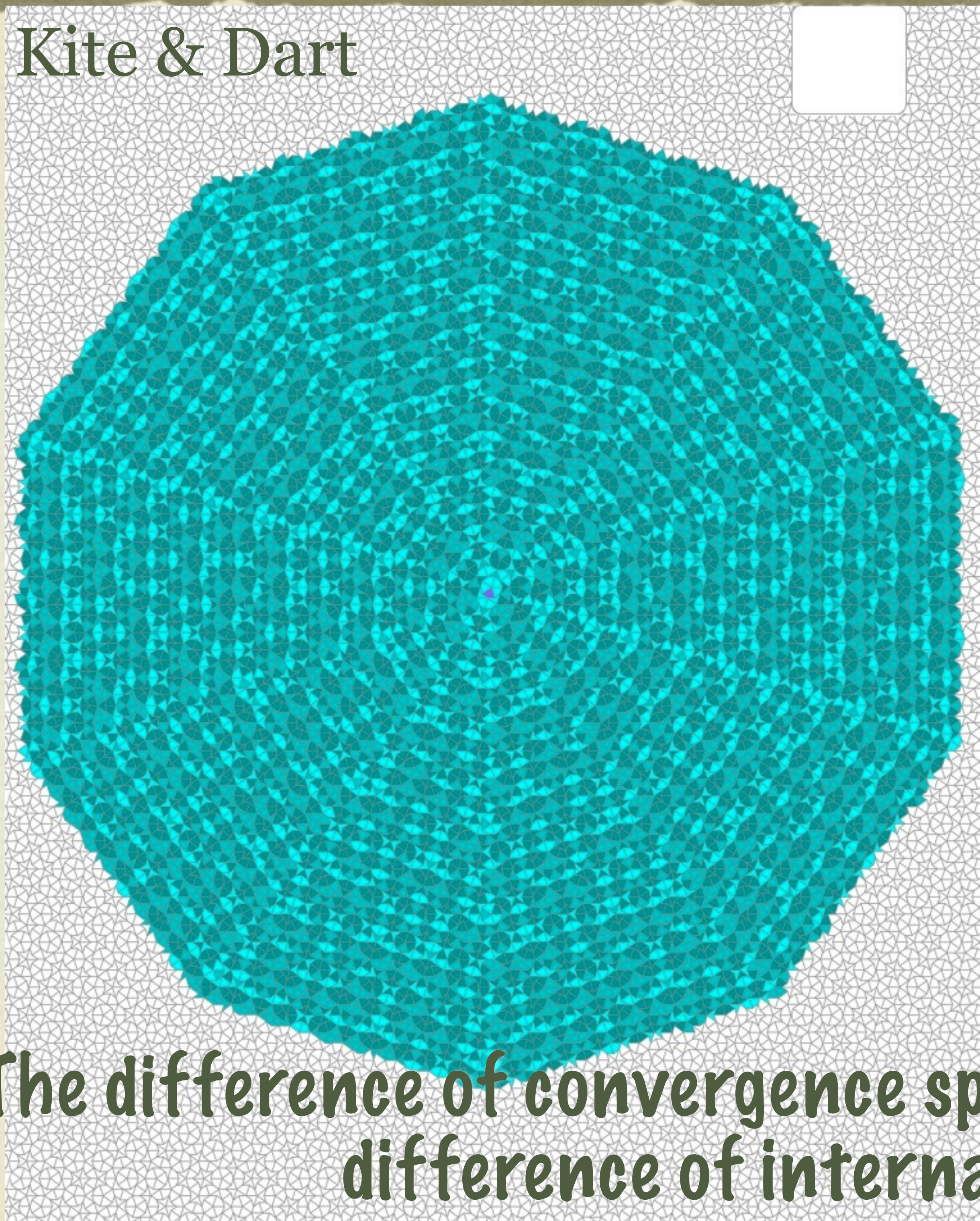



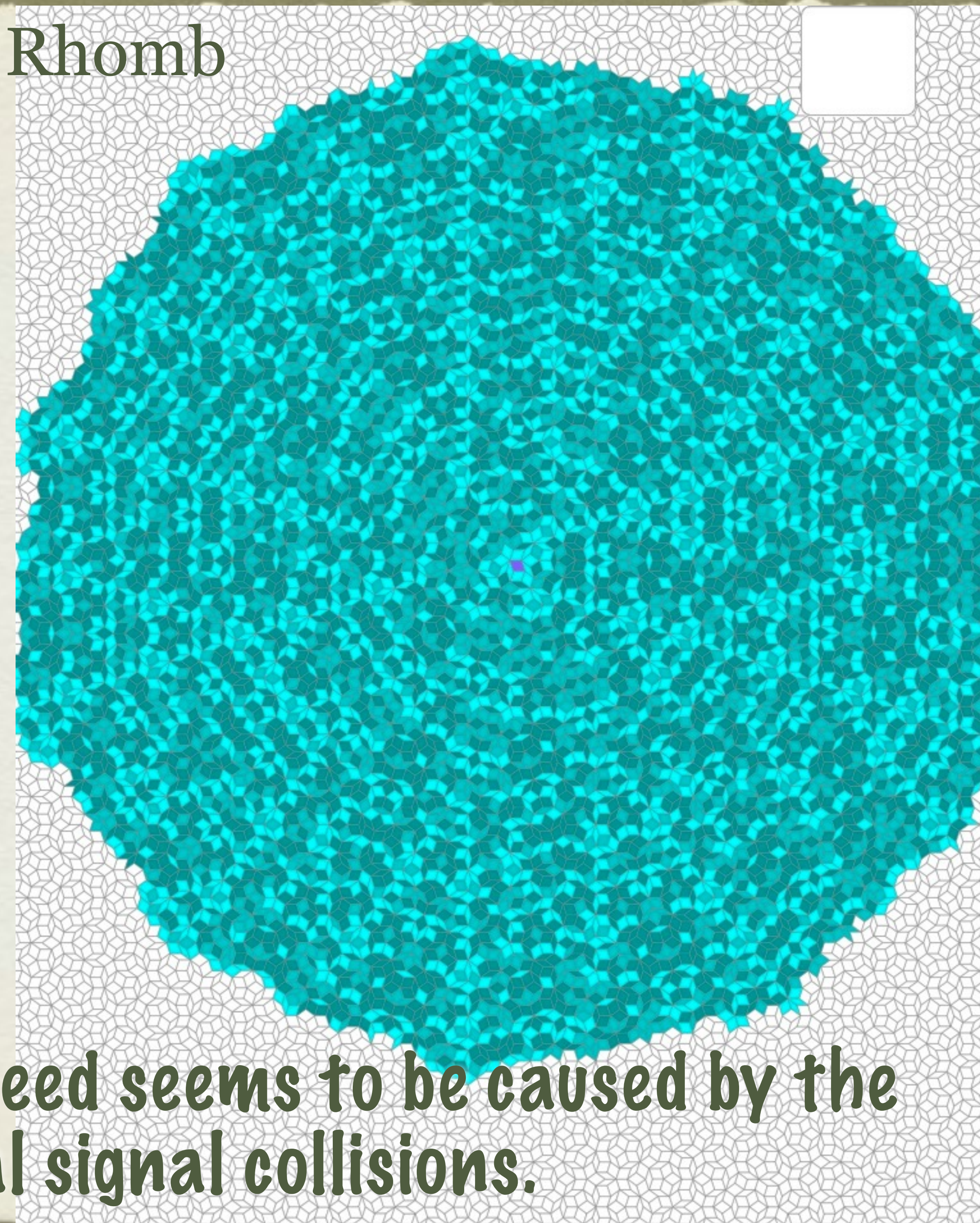
# VON NEUMANN



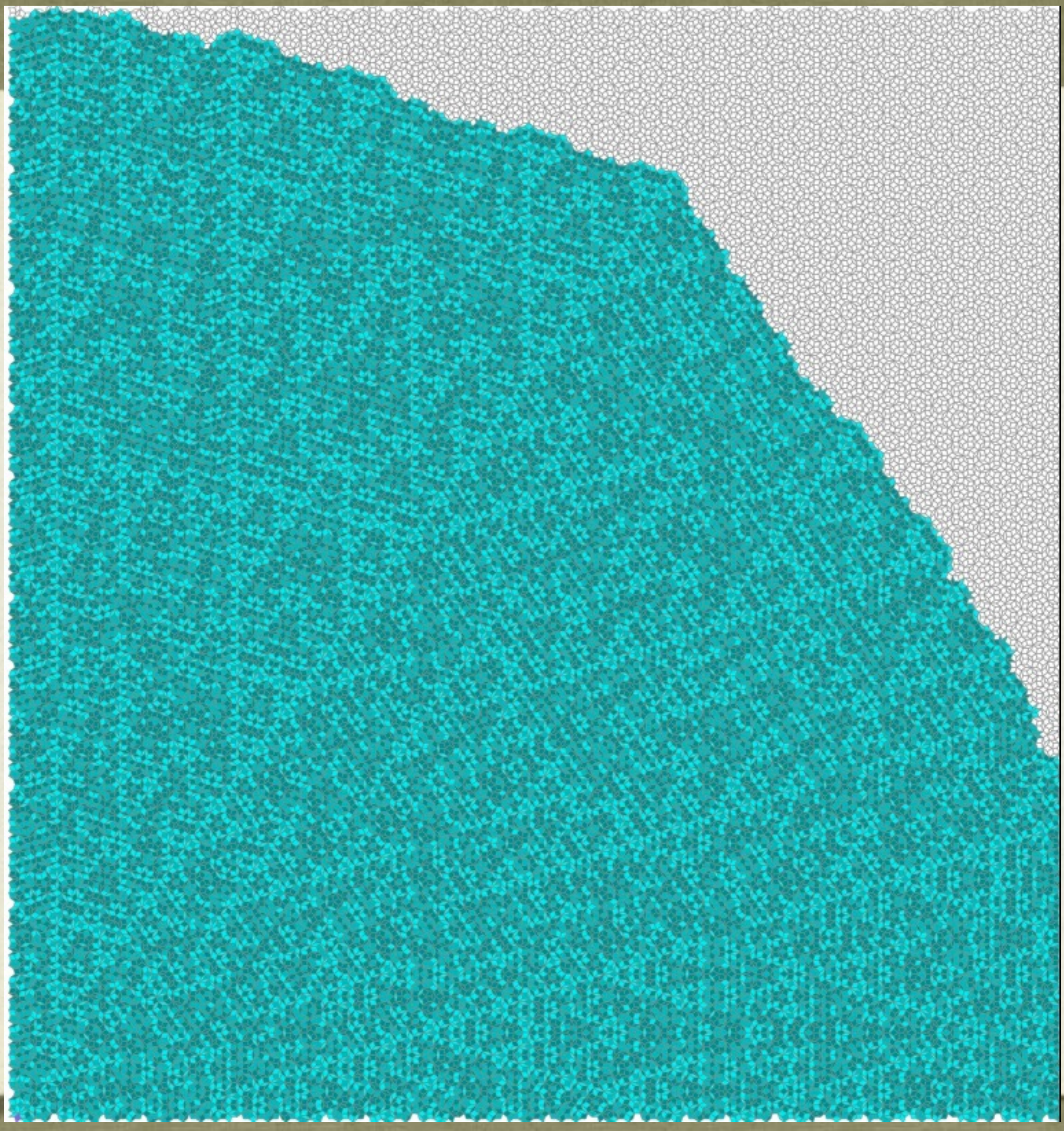
# MOORE




The difference of convergence speed seems to be caused by the difference of internal signal collisions.

# RHOMB, MOORE



The shape of its wavefront is composed by the complex signal collisions.

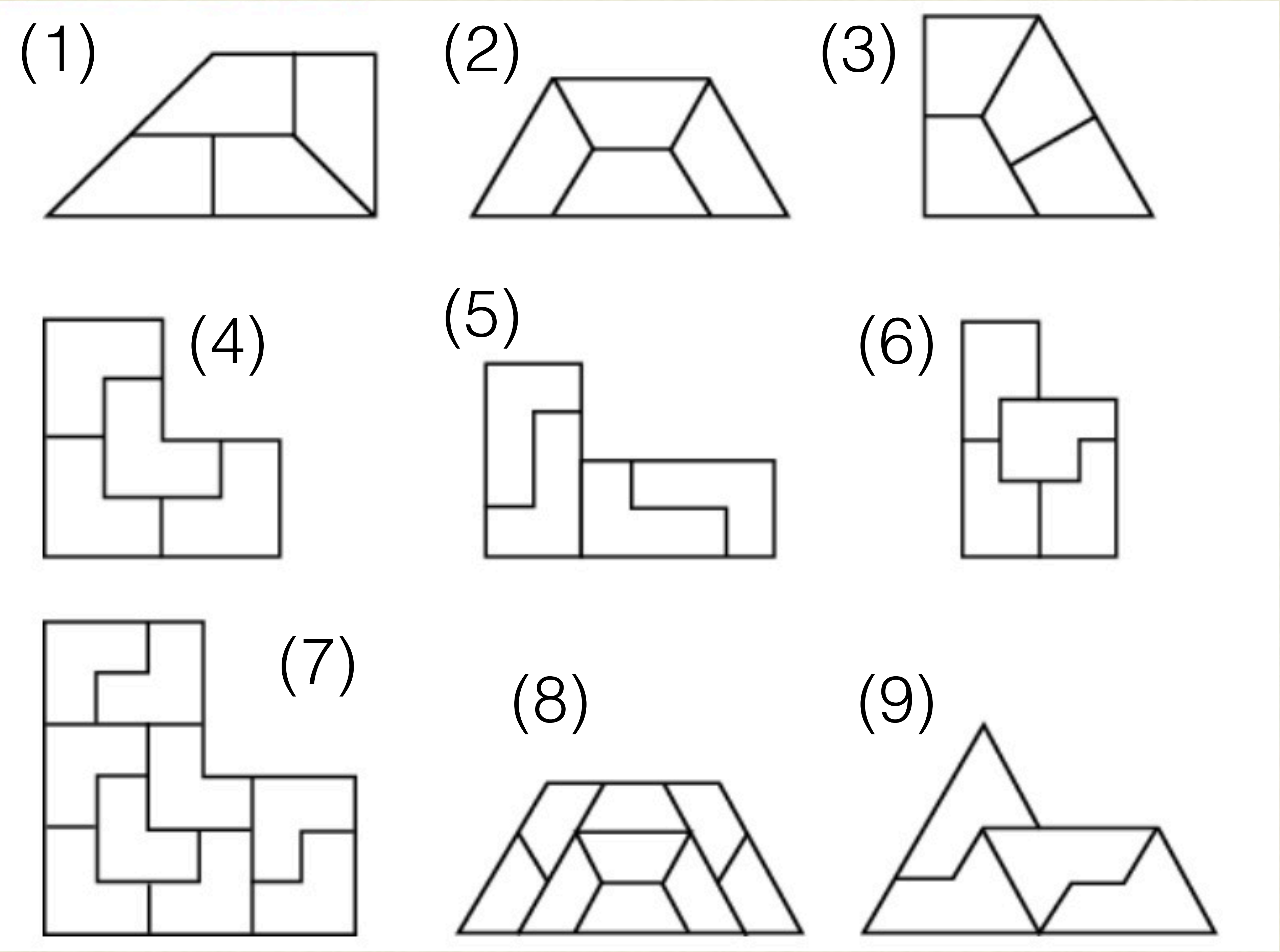
The complicated signal propagation and collision seems to depend on the aperiodicity of the tilings and chosen neighborhood. But...

cf. Wolfram class III CA.

- Thanks to the **Amman bars**, we can prove the corona limit of **Penrose tilings** is a **regular decagon** but it is a quite ad-hoc way.

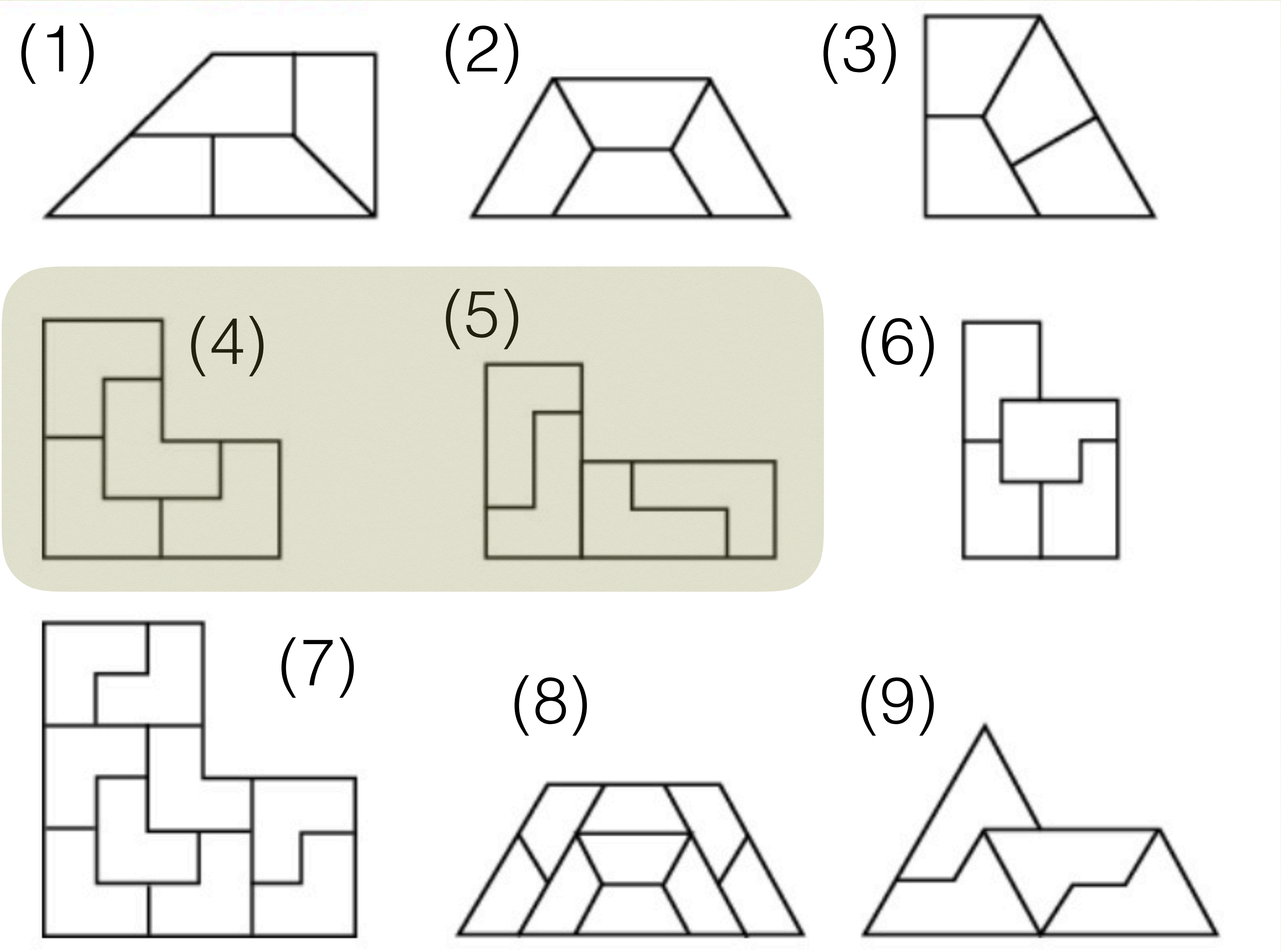
- **Ammann tiling**: another quasi-periodic tiling which has **Ammann bars**.
  - Its Ammann bar is even not parallel to the edge of corona limit.

# REP-TILES



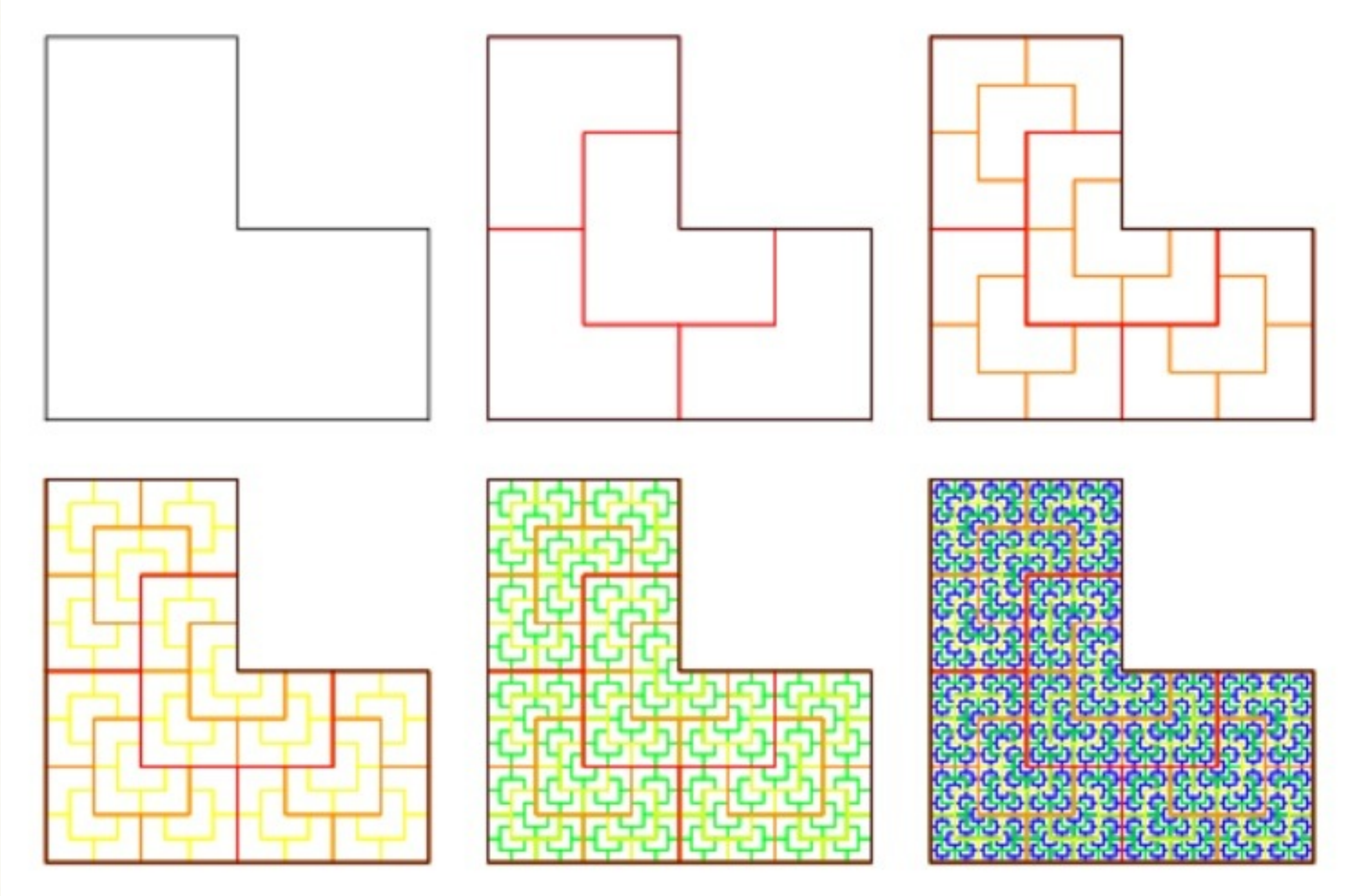

Wolfram MathWorld http://mathworld.wolfram.com/Rep-Tile.html

# REP-TILES




Wolfram MathWorld http://mathworld.wolfram.com/Rep-Tile.html

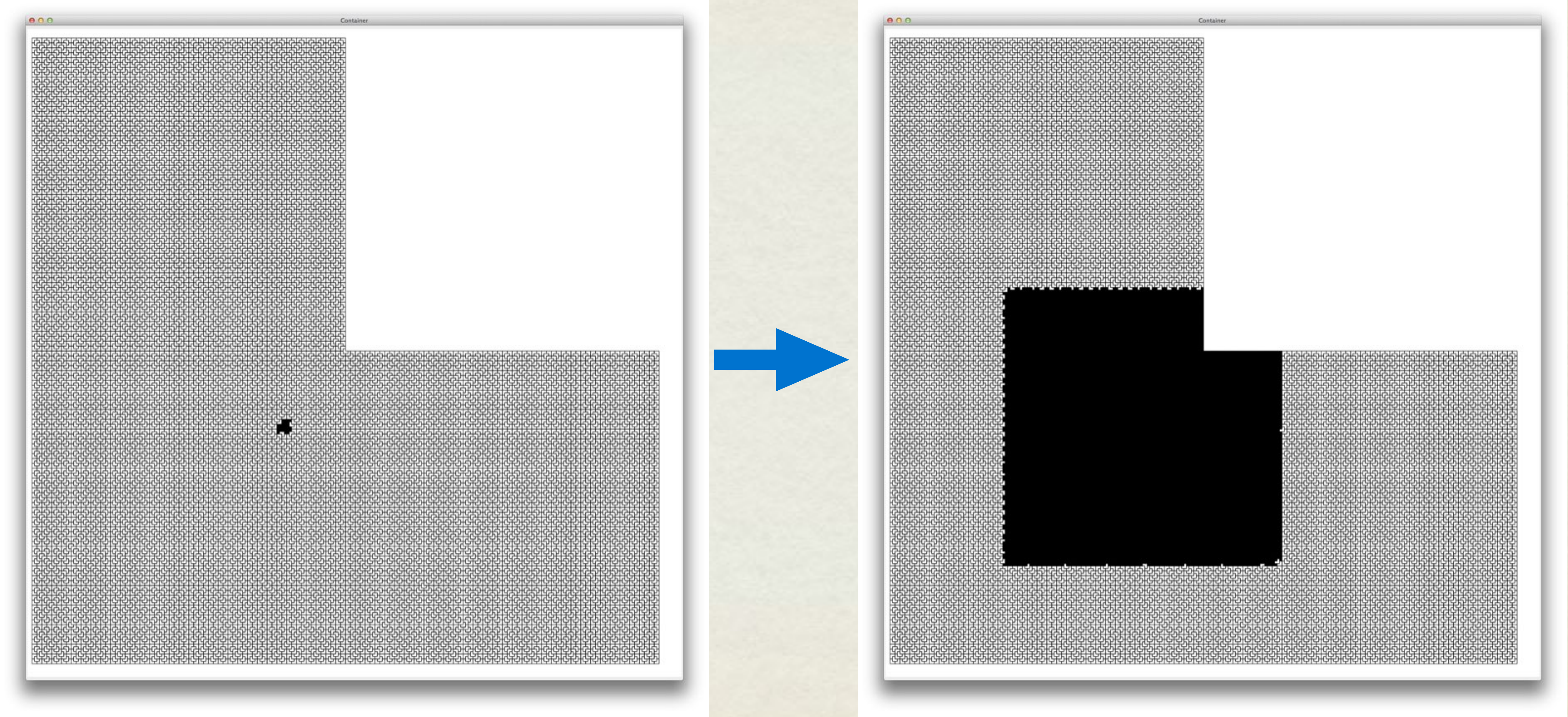
# REP-TILE #4



# CORONA: REP-TILE 4

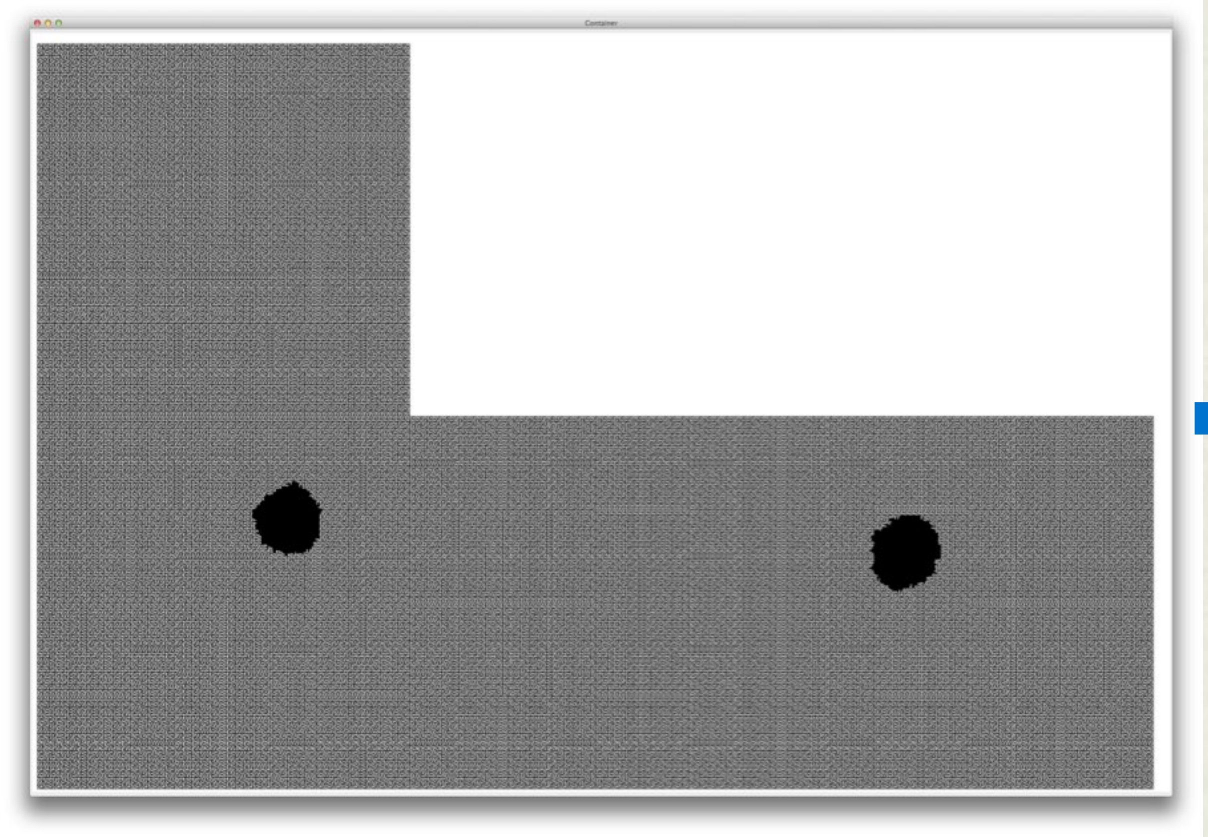
Square pattern

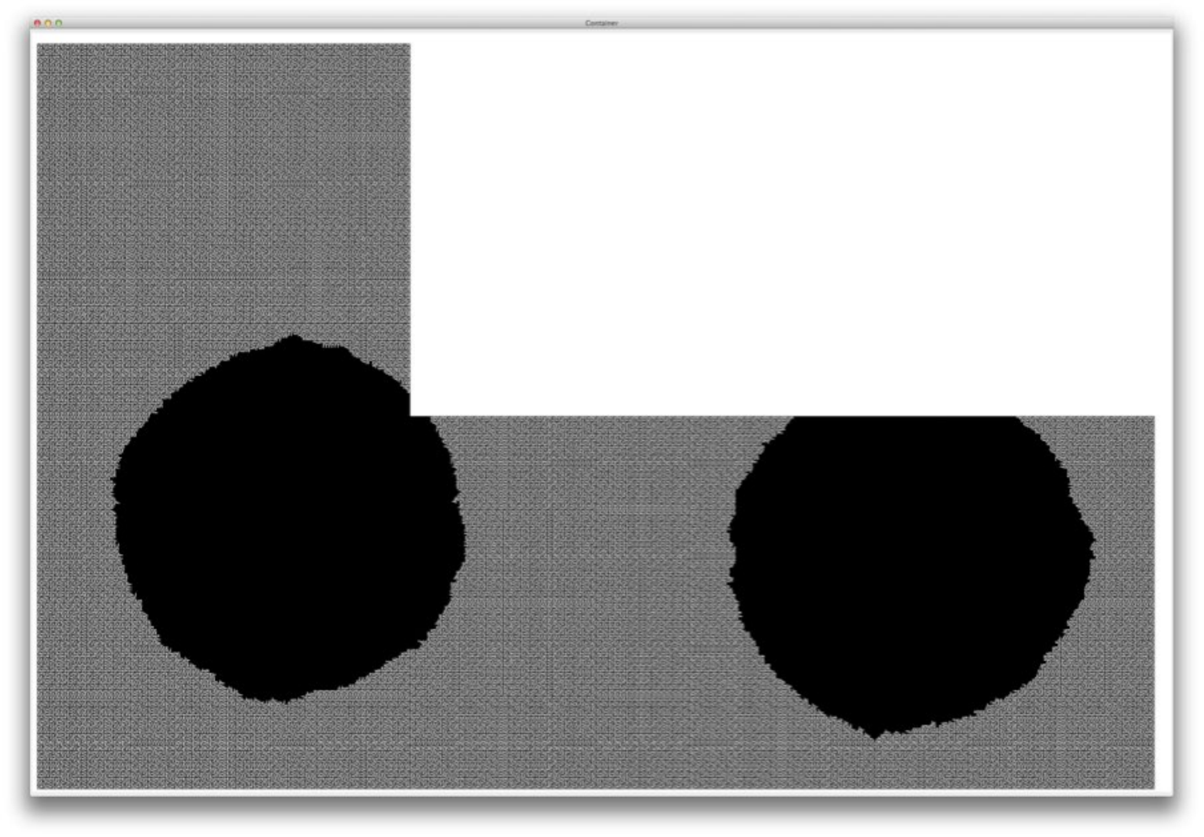
Rep-tile 4 has pure point spectrum.



# CORONA: REP-TILE 5

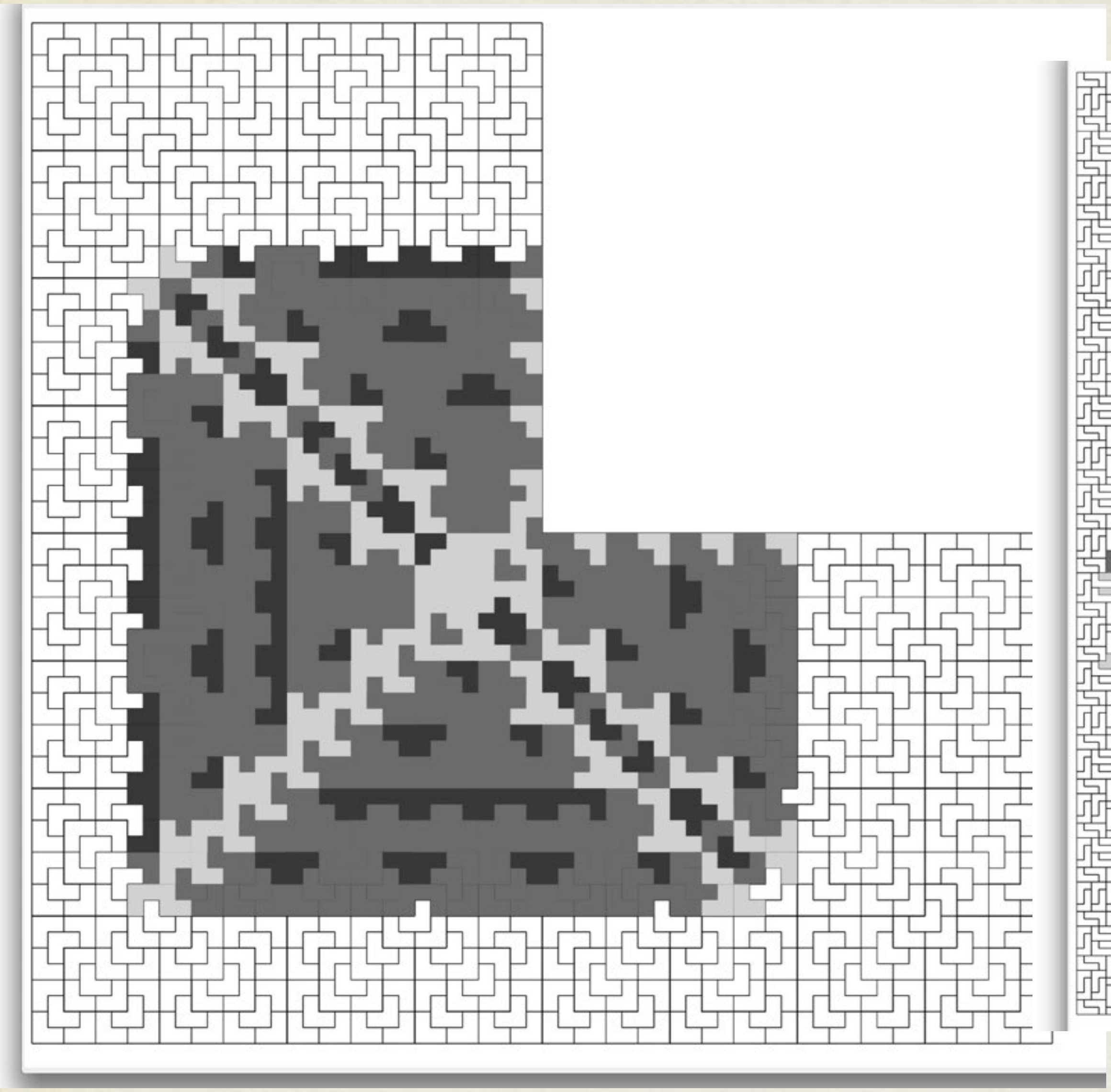
Its corona does not seem to converge a polygon...

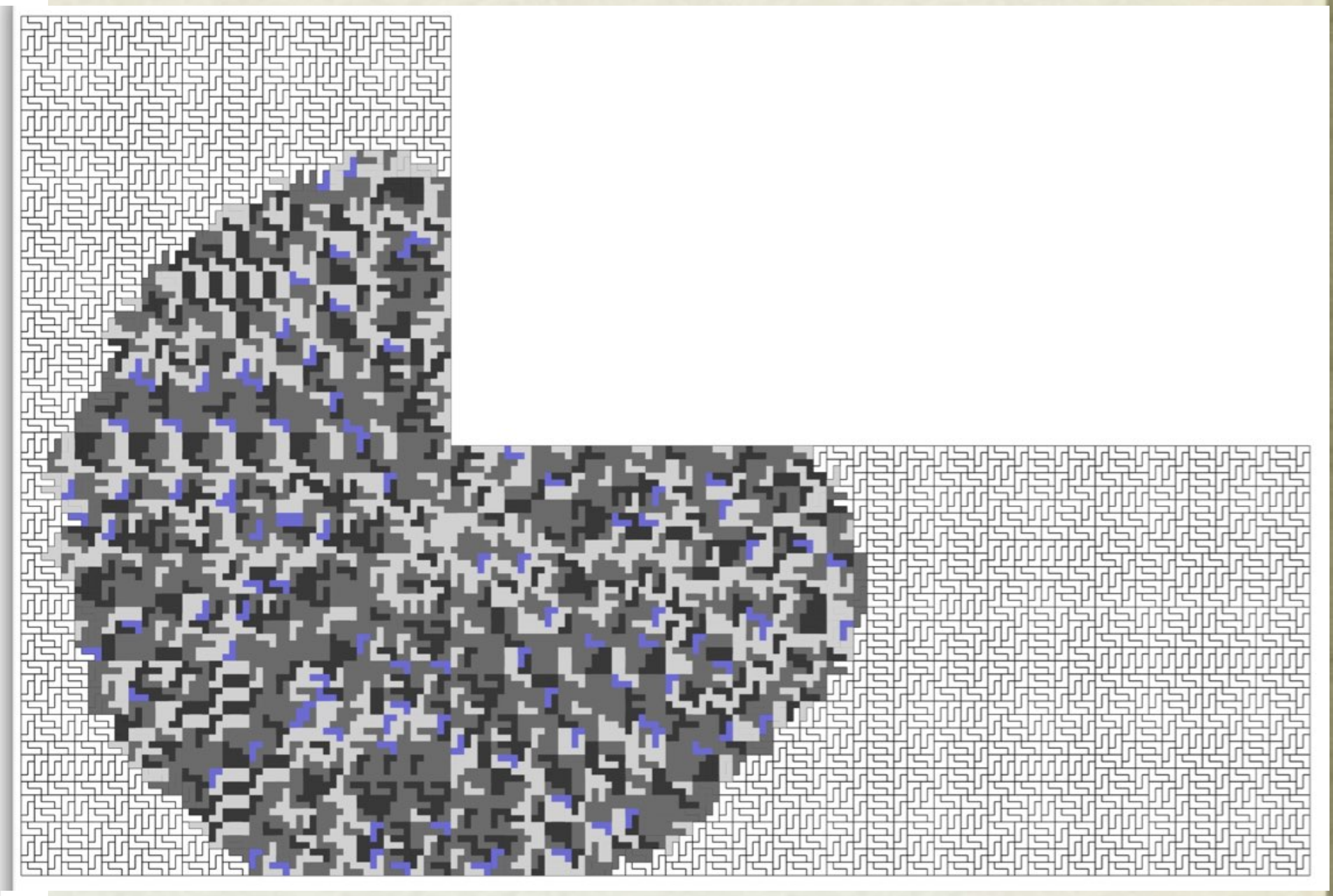




The spectrum of Rep-tile 5 contains continuous spectrum.

# REP TILE 4 & 5





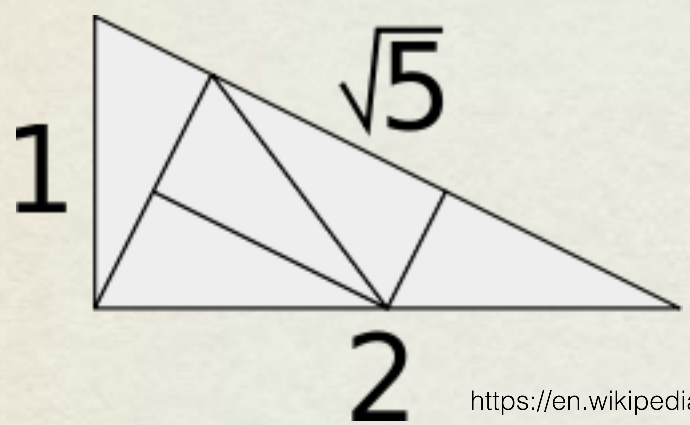
cf. Wolfram class IV CA.

The Fourier spectrum of Rep-tile 5 contains continuous.
It is not described by the similar projection method.

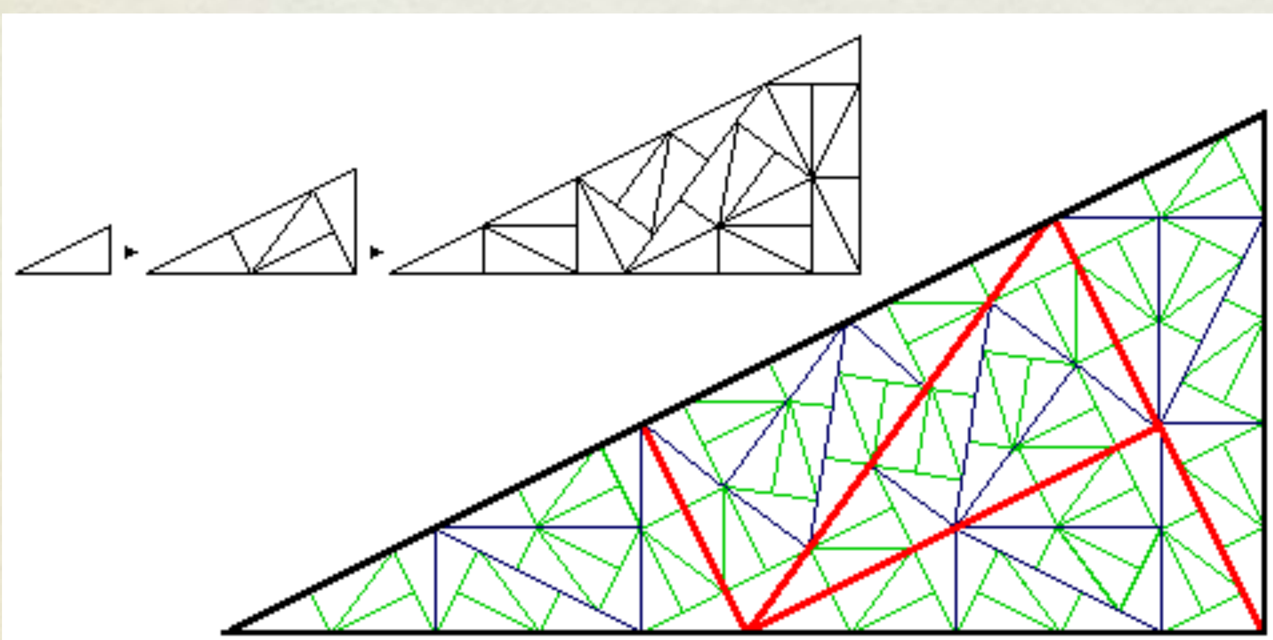
# PINWHEEL TILING

Triangular tiles appear in infinitely many orientations (dense, irrational angles).

Conway (unpublised), Radin 1994



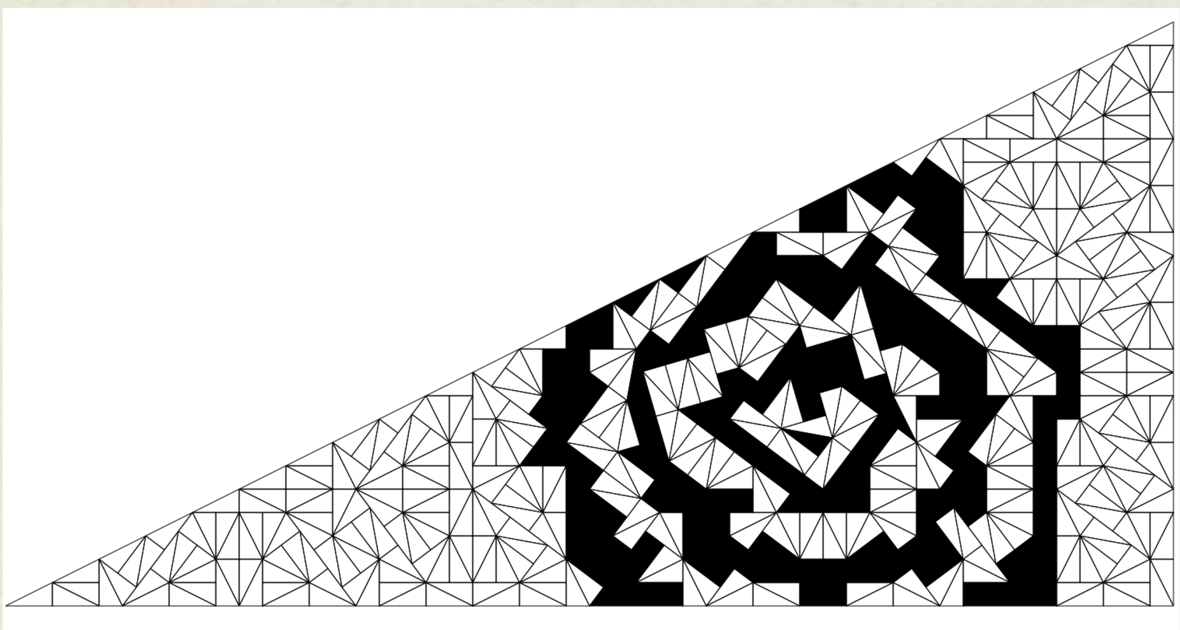

https://en.wikipedia.org/wiki/Pinwheel_tiling

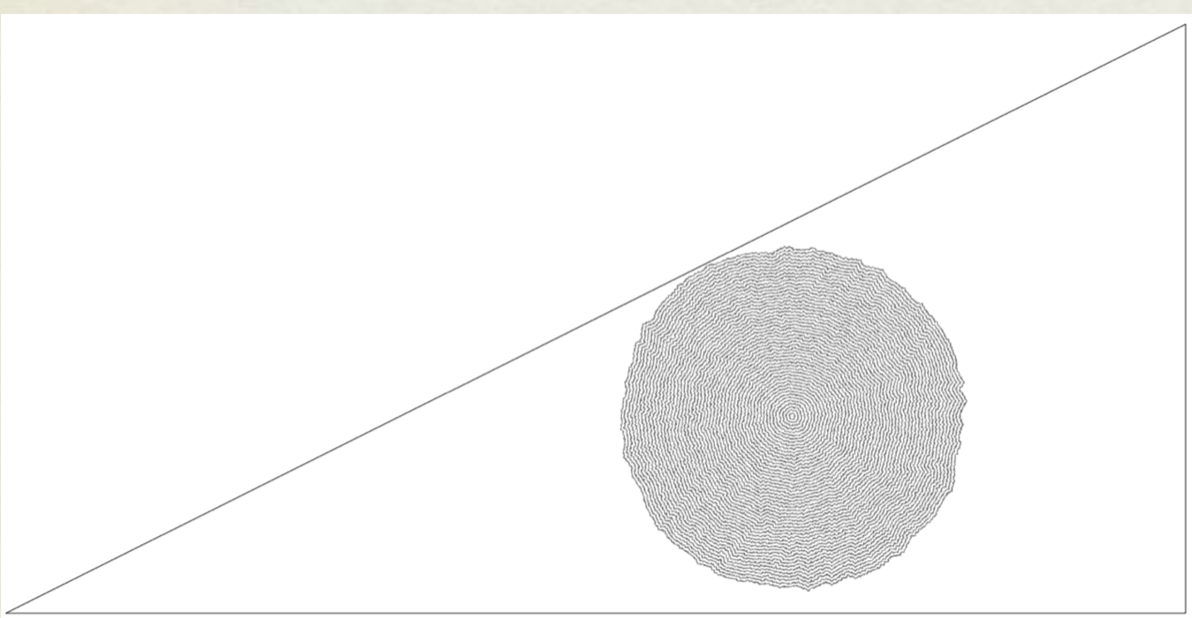


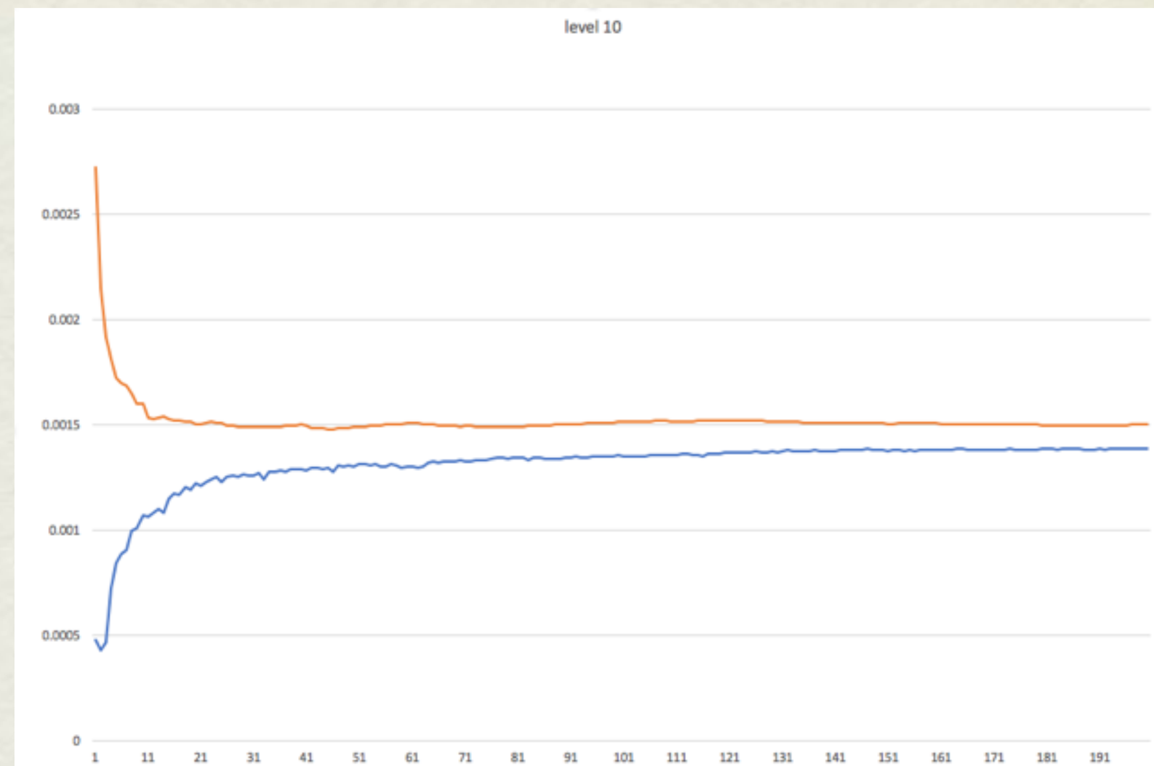
http://www.quadibloc.com/math/images/pinw14.gif

# CORONA LIMIT OF PINWHEEL TILING

The Corona of a pinwheel tiling








... a circle !?

# CORONA LIMIT OF PINWHEEL TILING

The corona limit of a pinwheel tiling is a **circle** of radius $\sqrt{5}$

Proof outline by Lorenzo Sadun
also cf. Radin, Sadun 1996

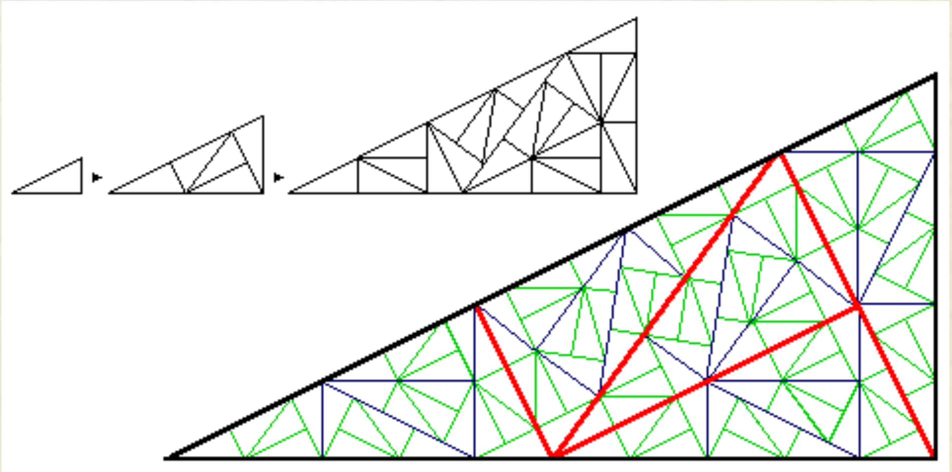
• A signal can propagate at speed √5 along the hypotenuse of an **even**-order supertile or along the short and long edges of an **odd**-order supertile.

• In the infinite-volume limit, there are high-order supertiles pointing in all directions — "**highways**"

• There is always a **nearby highway** heading more-or-less in the direction you want to go.

• In the **infinite limit**, the fraction of time spent **jumping from one highway to another goes to zero**.

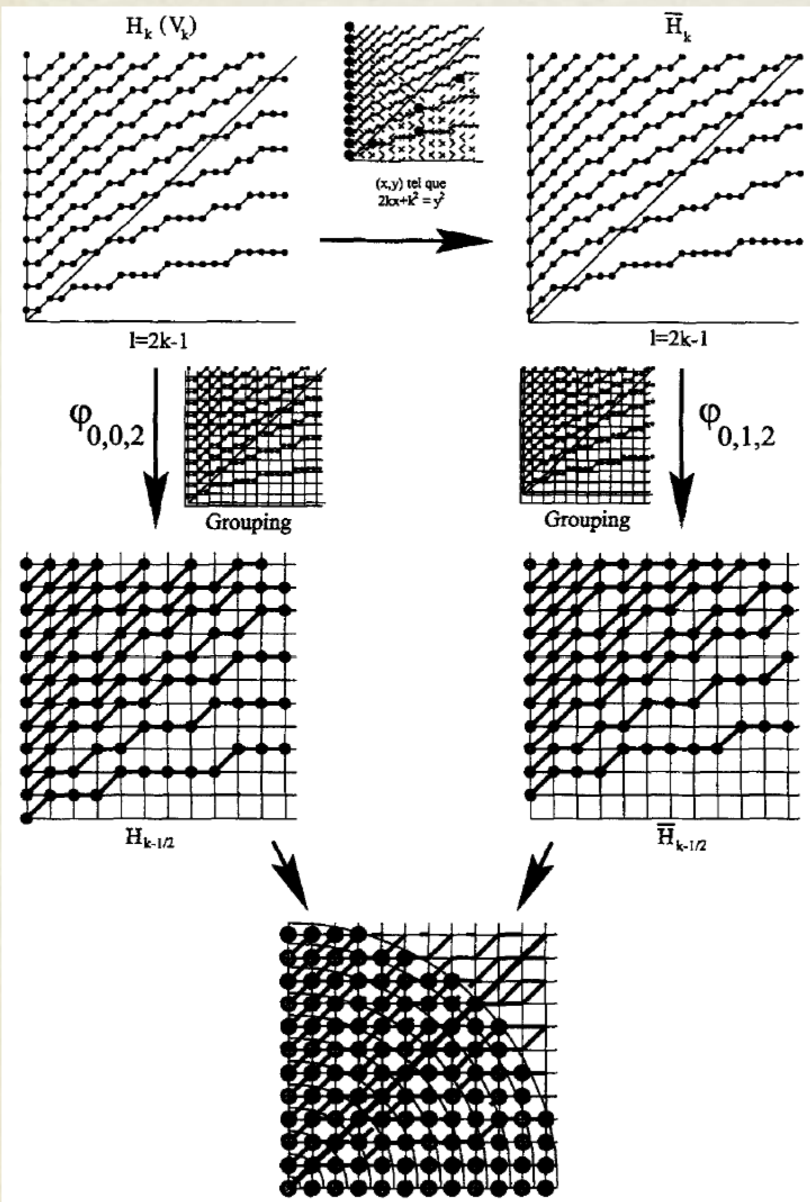
• the directions of the highways can be made arbitrarily precise, and the limiting speed in all directions is **exactly √5**.



But the convergence is **RIDICULOUSLY slow**!
The proof is only for this adjacency, but ....

# CELLULAR AUTOMATA GENERATING A CIRCLE

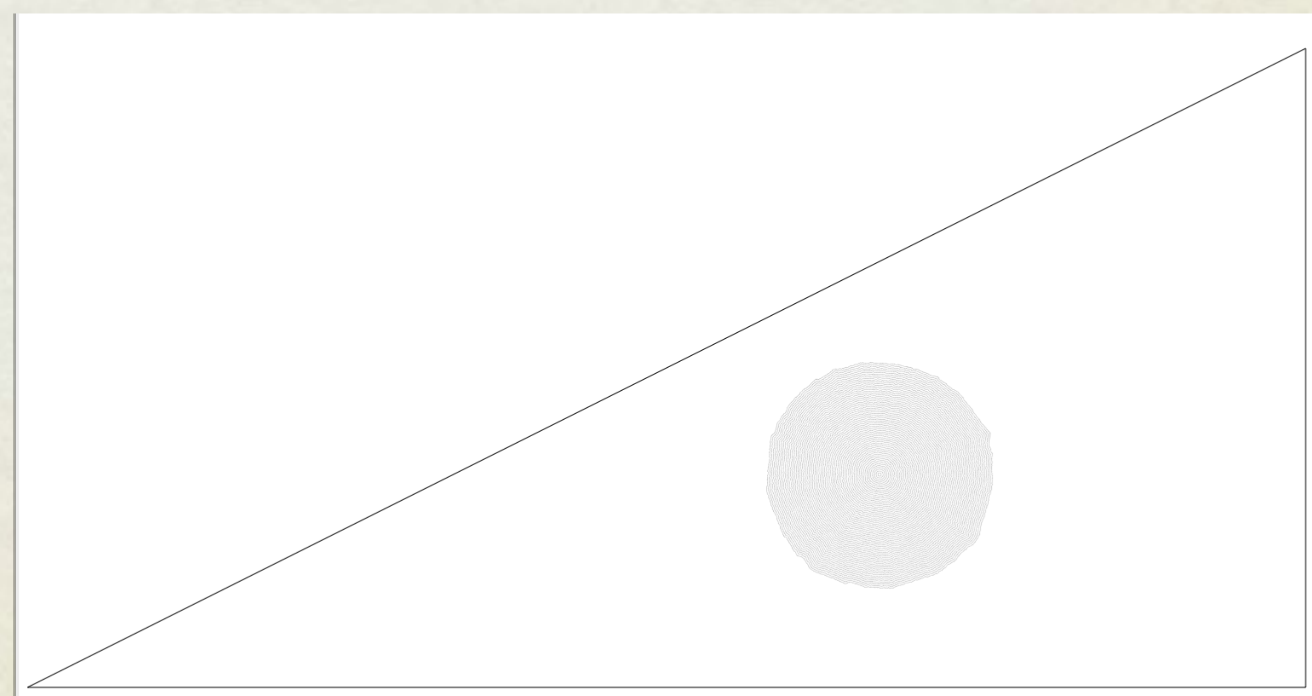
## A cellular automaton generating a circle




Delorme, Mazoyer, Tougne 1999
state number ~ 5^13

**A trade off: state number vs. tiling complexity**

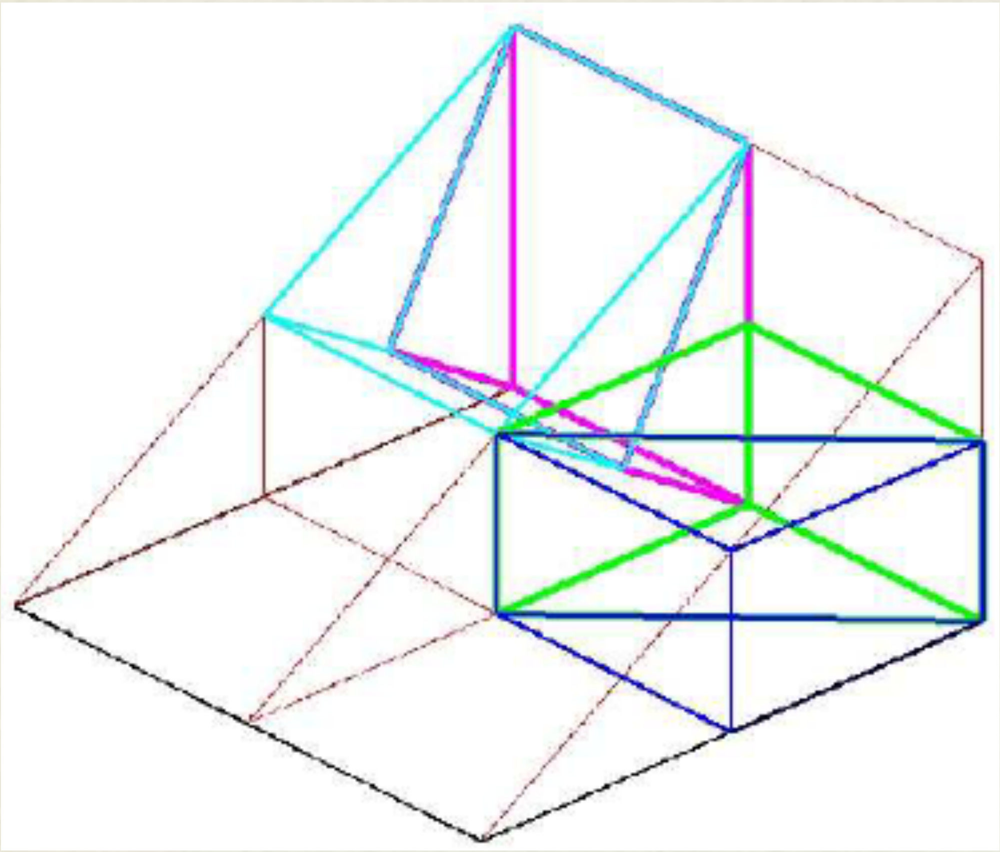
A simple diffusion CA on a pinwheel tiling
state number 2

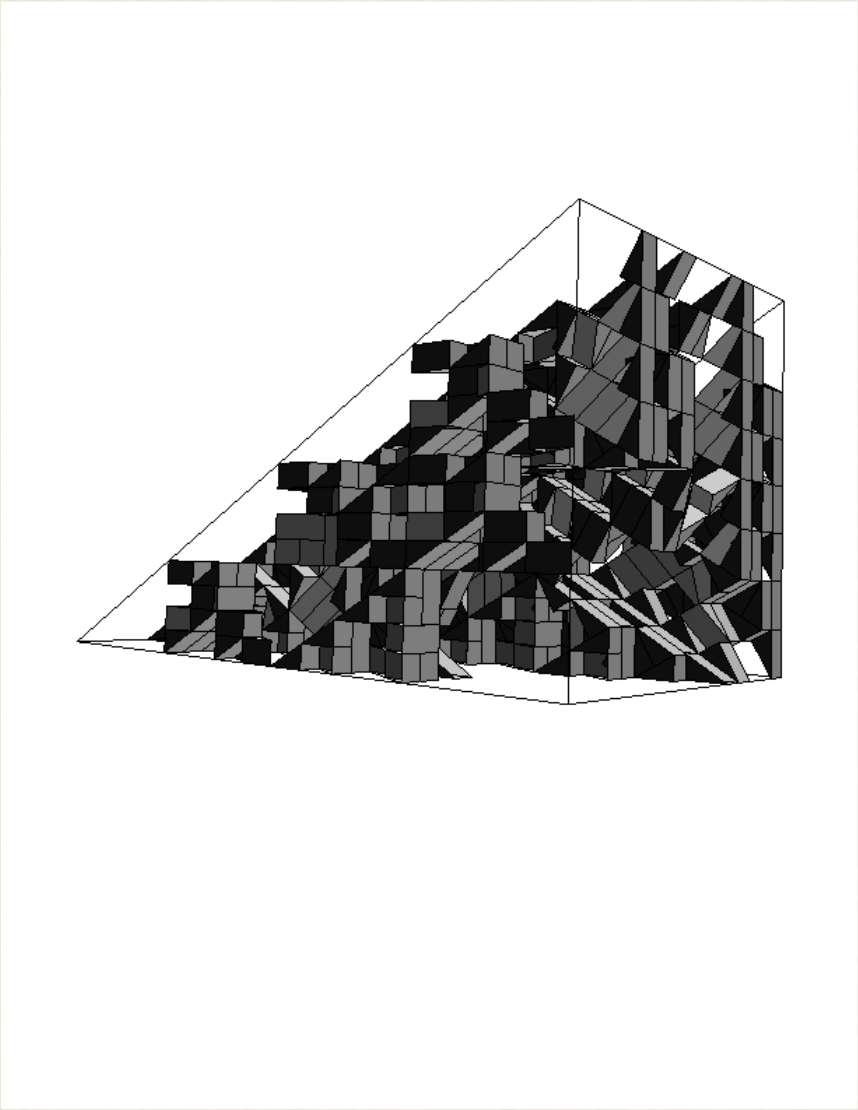


# SPHERICAL CORONA LIMIT??

Quaquaversal tiling: rotations are dense in SO(3).

Conway, Radin 1998





The corona limit of quaquaversal tiling might be sphere...??